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Broadcast Guidance of Multi-Agent Systems

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This paper is dedicated to the memory of *Professor Ioan Dzitac*, Alfred Bruckstein's former high school colleague at the Dragos Voda National College, in Sighetul Marmatiei, and a competitor at the Mathematical Competitions. It was wonderful to reconnect with him after many years, to recall the good old times, and to talk about future joint projects. His untimely death deeply saddened us all. His life and career are shining examples of our high school slogan: *"Per Aspera ad Astra"*. His legacy lives on.

Abstract

We consider the emergent behavior of a group of mobile agents guided by an exogenous broadcast signal. The agents' dynamics is modelled by single integrators and they are assumed oblivious to their own position, however they share a common orientation (i.e. they have compasses). The broadcast control, a desired velocity vector, is detected by arbitrary subgroups of agents,that upon receipt of the guidance signal become "ad-hoc" *leaders*. The control signal and the set of leaders are assumed to be constant over some considerable intervals in time. A system without "ad-hoc" leaders is referred to as autonomous. The autonomous rule of motion is identical for all agents and is a gathering process ensuring a cohesive group. The agents that become leaders upon receipt of the exogenous control add the detected broadcast velocity to the velocity vector dictated by the autonomous rule of motion. This paradigm was considered in conjunction with several models of cohesive dynamics, linear and non-linear, with fixed inter-agent interaction topology, as well as systems with neighborhood based topology determined by the inter-agent distances. The autonomous dynamics of the models considered provides cohesion to the swarm, while, upon detection of a broadcast velocity vector, the leaders guide the group of agents in the direction of the control.

For each local cohesion interaction model we analyse the effect of the broadcast velocity and of the set of leaders on the emergent behavior of the system. We show that in all cases considered the swarm moves in the direction of the broadcast velocity signal with speed set by the number of agents receiving the control and in a constellation determined by the model and the subset of "ad-hoc" leaders. All results are illustrated by simulations.

Keywords: multi-agent systems, broadcast guidance control, linear and nonlinear dynamic systems, emergent behavior

1 Introduction

Consider a group or swarm of mobile agents and a controller that broadcasts a guidance signal, referred to as the *exogenous control*, aimed at guiding the swarm in a desired direction. The agents move in the plane and the exogenous control is a velocity vector. The velocity signal sent by the controller is detected by an arbitrary subset of agents, that become induced *leaders*. Agents that do not detect the exogenous control are referred to as *followers*. All agents, leaders and followers, apply a gathering inter-agent interaction rule of motion.

The agents are *identical*, modelled by single integrators and are assumed oblivious to their own location in the plane. The agents do not have explicit communication capabilities, however each agent can sense information about some or all other agents in the swarm and possibly detect the exogenous control. All agents autonomously behave according to identical pre-determined rules of motion based on the sensed information. We treat a variety of autonomous interaction models, both linear and non-linear, leading to convergence to a common position (gathering). The gathering properties of the autonomous models have been widely treated in the literature. The main contribution of the present work is to investigate the impact of a broadcast velocity signal, detected by an arbitrary set of leaders, on these systems.

An agent whose relative information is available to an agent i is referred to as a *neighbor* of i and the set of neighbors is referred to as the *neighborhood* of i. The neighborhood of each agent may be pre-determined and position independent, therefore time invariant, or determined by the relative location of the agents. The topology of information exchange between agents is represented by a graph.

In the present work we consider two types of relative location information, due to the sensing processes that may be available to the agents:

1. Bearing and distance, i.e. relative position of neighbors
2. Bearing only to neighboring agents

The sensing of neighbors by each agent yields information in its local frame of reference. The orientation of all local coordinate systems is assumed to be aligned to that of a global coordinate system, i.e. by assumption all agents have a compass but no access to a Global Positioning System (GPS).

In the existing literature leaders are pre-defined agents with additional knowledge, such as current position, goal position or possibly even different and predefined dynamics. Sometimes they are special agents added to the swarm with the purpose of controlling the collective behavior. For example

1. *Leaders that are not influenced by other agents in the system* are discussed in [11], [24], [16], [15], [17]. The leaders are either fixed in the environment or follow a predefined movement
2. *Shills*[1] *and soft control* are treated in [9],[10]. Shills are assumed to know the moving rules and the current state of the regular agents and plan their own movement rules such that a desired goal is achieved.
3. *Leaders combining the consensus protocol with goal attraction* are considered in [7], [8], where a predefined goal position is known only to the leaders. Leaders' dynamics includes a goal attraction term which is a function of the leader's distance from the goal position. Thus, such leaders are assumed to be aware of their own position.

In our paradigm leaders are not special agents, neither predefined nor with special capabilities. Any agent can become a leader and the system can have any number of leaders. Moreover, the subset of leaders may change in time. The leaders abide by the same autonomous dynamics as the remaining agents, with the detected velocity signal added by the leaders to their dynamics. To the best of our knowledge this is a novel paradigm.

This model is applied to various **autonomous** dynamics, linear and non-linear, as well as to various topologies, fixed as well as position dependent.

[1](A Shill is a decoy who acts as an enthusiastic customer in order to stimulate the participation of others)

1. ***Linear Dynamics and predefined topology***

   (a) **Linear with Uniform influence**: The velocity of each agent is determined by the sum of its relative position to its predefined and time independent neighbors. The topology links are bi-directional.

   (b) **Linear with Scaled influence**: The velocity of each agent is determined by the sum of its relative position to its predefined and time independent neighbors, scaled by the inverse of the number of neighbors. The topology links are bi-directional.

   (c) **Linear Cyclic Pursuit.** The agents, in a group of n agents, are labeled from 1 to n and the dynamics of agent i is determined by its relative position to agent $(i \mod n) + 1$, referred to as its *successor*. The information flow is directed from successor to agent.

   (d) **Deviated Linear Cyclic Pursuit.** Same as in Linear Cyclic Pursuit, except that the direction of movement of agent i is deviated from its line of sight to agent $(i \mod n) + 1$ by an angle θ, common to all agents.

2. ***Position dependent topology, due to finite visibility***
   In these models the agents are assumed to have myopic sensors, also known as sensors with finite visibility range, denoted by R. An agent j is a neighbor of agent i if and only if the geometric distance between the two is less than or equal to R.

   (a) **"Piecewise Linear" with Uniform Influence**. Due to finite visibility, links between agents may connect or disconnect when the agents enter/exit the visibility range. Hence, the interaction graph(s) may connect and/or disconnect. In-between topology changes each connected component behaves as in the case of Linear dynamics with Uniform influence and Fixed Topology.

   (b) **Potential Based Dynamics.** In this *non-linear* model, the velocity of each agent is given by the derivative of a bounded distributed potential function defined around the position of the agent. The potential function is designed such that the resulting motion preserves connectivity. The main idea is to design the potential function such that when two agents are about to lose connectivity, the gradients of their corresponding potential fields lie in the direction of the edge connecting the two agents, aiming to shrink it, see [1], [2]. Navigation-like distributed potential functions have been shown to autonomously *ensure convergence while preserving connectivity.*

   The main purpose of this part of our work is to design an exogenous control such that these properties are preserved, while causing the swarm to drift in the desired direction.

3. ***Non-linear models with possible topology changes caused by agent capture and merge processes***

   (a) **Bearing-only Cyclic Pursuit.** The topology of the information flow here is the same as for the case of linear cyclic pursuit, but the agent senses is only the direction to the agent being chased, refered to as its successor. The velocity of each agent has magnitude 1 and its direction is the line of sight to its successor. In this model, an agent may catch up with its successor and be captured by it. In this case the agents merge and inherit the dynamics of the successor. Moreover, the merged agent is a leader if and only if the captured and/or the capturing agents have been leaders.

   (b) **Deviated Bearing Cyclic Pursuit.** As in Bearing-only Cyclic Pursuit, except the direction of movement of agent i is deviated from the line of sight to its successor by an angle θ, common to all agents. Here too, if an agent catches up with its successor, it merges with it and both move with the dynamics of the successor. With exogenous control, if either or both the captured agent and its successor previously detect the control, then after capture the combined agent will be driven by the exogenous control too.

We show the impact of the broadcast velocity signal and of the subset of leaders on each of the above models.

1.1 Paper Outline

- In **section 2**, we consider a system where the autonomous velocity of each agent is the sum or weighted sum of its relative position to its neighbors. The former model is referred to as the *Uniform Influence* model, the latter, when the sum is weighted by the inverse of the degree of the agent, is referred to as the *Scaled Influence* model.
- In **section 3** we investigate the emergent behavior of a system under Cyclic Pursuit. We consider the Relative Position Model, as well as the Bearing Only Information Model.
- In **section 4** we investigate the emergent behavior of the system under Deviated Cyclic Pursuit, linear and non-linear (Bugs) whereby the direction of autonomous movement of each agent is deviated by an angle θ from the line of sight to its successor. The leaders add the detected exogenous velocity to their autonomous velocity. The angle θ is common to all agents.
- In **section 5**, we treat systems with finite visibility, where an agent j is a neighbor of agent i if and only if the geometric distance between the two is less than or equal to the visibility range, R.
  - We first investigate the behavior of a **Piecewise Linear System** with Uniform Influence and time invariant parameters. When agents have finite visibility, links connect and disconnect when the distance between agents reaches the visibility range from above and below respectively. Moreover, a link connection or disconnection may result in graph connection or disconnection.
  - We then treat the **Potential Based Model** with distributed navigation-like bounded potential functions. In this model, the autonomous velocity of agents is the gradient of a local potential. Applying a navigation-like distributed potential function ensures, in autonomous systems, gathering at a common position while preserving all initial links ("never lose neighbors"). We investigate the impact of the exogenous guidance control on the behavior of the system.

1.2 Notations

In the sequel, i denotes an agent, $i = 1, \ldots, n$; n is the number of agents; $n_l(t)$ is the number of leaders at time t; $p_i(t) = [x_i(t) \quad y_i(t)]^T$ is the position of agent i at time t; $N_i(t)$ is the set of neighbors[2] (i.e. the neighborhood) of i at time t; $d_i = |N_i|$ is the number of neighbors of i, degree of agent i; $U(t) = [U_x(t) \quad U_y(t)]^T$ is the broadcast velocity vector; $b_i(t) = 1$ if i is a leader at time t, 0 otherwise; $B =$ vector of leaders, $B(i) = b_i$

2 Uniform and Scaled Linear Dynamics

2.1 The Model

In this model we consider a group of n agents moving in $\mathbb{R}^2$, where the motion of agent i is given by

$$\dot{p}_i(t) = \sum_{j \in N_i(t)} \sigma_{ji}(p_j(t) - p_i(t)) + b_i(t)U(t) \tag{1}$$

where σ_{ji} quantifies *the strength of the influence* of agent j on the movement of agent i.

In the present section we investigate the behavior of the system under the following assumptions:

- *time-invariant exogeneous parameters* - the exogeneous control and the set of leaders are fixed in time $U(t) = U; b_i(t) = b_i; \ \forall t$
- *time-invariant connectivity* - the set of agents that affect the dynamics of any agent i is fixed, i.e. $N_i(t) = N_i$

[2] Agent j is a neighbor of agent i if agent i has information about j

Two models of influence are considered:

1. *Uniform* - The influence of all neighbors on any agent is identical , i.e. $\sigma_{ji} = 1;\ \forall j \in N_i; \quad i = 1, \ldots, n$.
2. *Scaled* - The influence of an agent $j \in N_i$ on i is scaled by the size of the neighborhood N_i, i.e. for each i, we have $\sigma_{ji} = \dfrac{1}{d_i}; \forall j \in N_i; \quad i = 1, \ldots, n$, where $d_i = |N_i|$, the number of neighbors of i.

In both models we assume that $j \in N_i$ if and only if $i \in N_j$ for all i, j.

Under the assumptions above, the position axes in (1) can be decoupled, i.e. $x_i(t)$ and $y_i(t)$ can be considered separately

Let $x(t) = (x_1(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ and $B = (b_1, \ldots, b_n)^T \in \mathbb{R}^n$. Then we have

$$\dot{x}(t) = -Lx(t) + B\,U_x \tag{2}$$

where L is the Laplacian of the system and similarly for $y(t)$. The solution of Eq. (2) is (ref. ([12])

$$x(t) = e^{-Lt}x(0) + \int_0^t e^{-L(t-\tau)} B U_x \mathrm{d}\tau \tag{3}$$

Using the properties of the Laplacian (see Appendix (7.1) we obtain the asymptotic behavior of $x(t)$ as

$$x(t \to \infty) = \alpha_x\, \mathbf{1}_n + (\beta\, U_x\, \mathbf{1}_n)\, t + \boldsymbol{\varrho}\, U_x \tag{4}$$

and similarly for $y(t \to \infty)$.

2.2 Asymptotic position in $\mathbb{R}^2$

Recalling that $p_i(t) = (x_i(t), y_i(t))^T$ we obtain

$$p_i(t \to \infty) = \alpha + \beta U t + \varrho_i\, U \tag{5}$$

where $\alpha = [\alpha_x \;\; \alpha_y]^T$ and $U = [U_x \;\; U_y]^T$ and the values of α_x, α_y, β and $\boldsymbol{\varrho}$ for the two Influence models are summarized in Table (1).

Table 1: Coefficients of asymptotic position

	Uniform influence	**Scaled influence**
α_x	$\frac{1}{n}\mathbf{1}_n^T x(0)$	$\frac{\mathbf{d}^T x(0)}{\sum_{i=1}^n d_i}$
α_y	$\frac{1}{n}\mathbf{1}_n^T y(0)$	$\frac{\mathbf{d}^T y(0)}{\sum_{i=1}^n d_i}$
β	$\frac{n_l}{n}$	$\frac{\sum_{i \in N^l} d_i}{\sum_{i=1}^n d_i}$
$\boldsymbol{\varrho}$	$\sum_{k=1}^{n-1} \left[\frac{1}{\lambda_k^u} V_k^u (V_k^u)^T\right] B$	$\sum_{k=1}^{n-1} \left[\frac{1}{\lambda_k^s} V_k^s (W_k^s)^T\right] B$

The asymptotic moving position of agents can be described as follows:

- The agents align along a straight line in the direction of U, anchored at the centroid of the initial positions $\alpha = [\alpha_x\,,\, \alpha_y]$.
- The velocity of all agents is $\beta\, U$. where
  - for the uniform case β depends only on the number of leaders
  - for the scaled case β depends on the subset of leaders
- $\boldsymbol{\varrho}$ determines the spread of the agents along the line of motion and is computed from the spectral representation of the system Laplacian, $L = V \Lambda W^T$ (see Appendix (7)

2.3 Simulation results

In this section we illustrate the impact of the Influence Model and of the leader selection on the agents dynamics by an example of 5 agents with initial positions and topology as in Fig. (1).

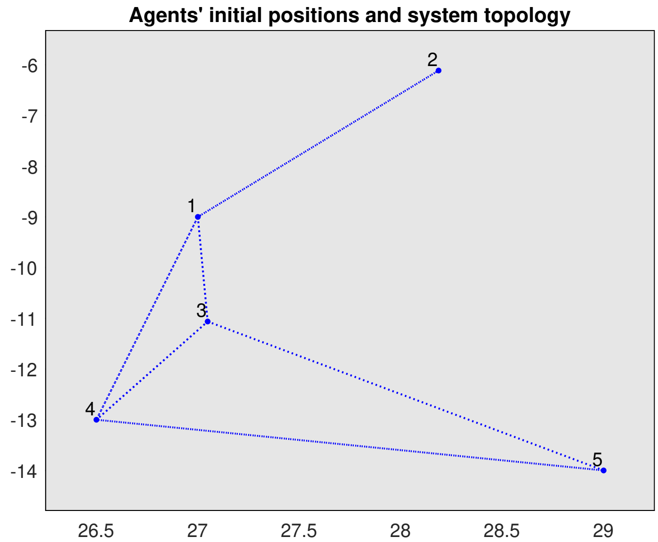

Figure 1: Topology and initial positions - Example 2.1

Example 2.1. *We first assume one leader, agent 2. Fig. (2) shows the emergent dynamics with Uniform Influence. Fig. (3) depicts the emergent dynamics with Scaled Influence. In both cases, the agents align asymptotically along a line with slope 0.2, equal to the slope of* $U = (5, 1)$.

Fig. (4) and (5) depict agents' velocities with Uniform and Scaled Influence respectively. The collective speed of the agents with scaled influence is considerably lower than that of the agents with uniform influence. While the asymptotic collective velocity in the uniform case is $0.2U$*, corresponding to* $\beta = 1/5$*, for the scaled case it is only* $0.0833U$*, corresponding to* $\beta = \dfrac{\sum_{i \in N^l} d_i}{\sum_{i=1}^{n} d_i}$.

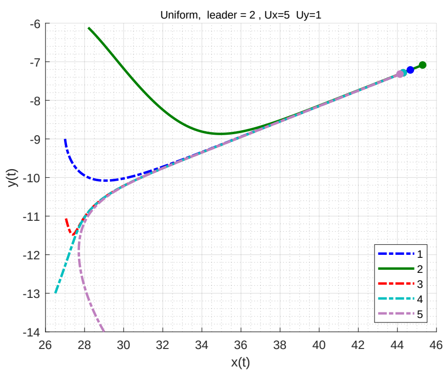

Figure 2: Emergent trajectories - Example 2.1 with Uniform Influence

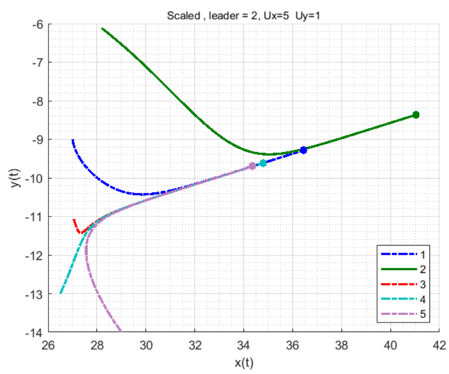

Figure 3: Emergent trajectories - Example 2.1 with Scaled Influence

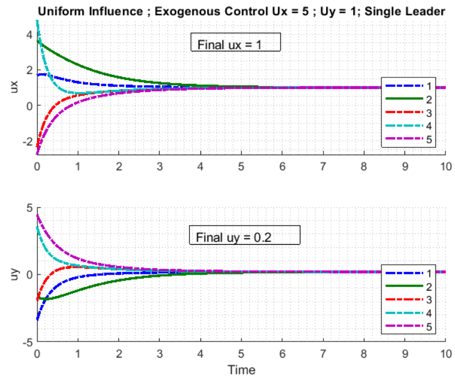

Figure 4: Velocities for Uniform Influence - Example 2.1

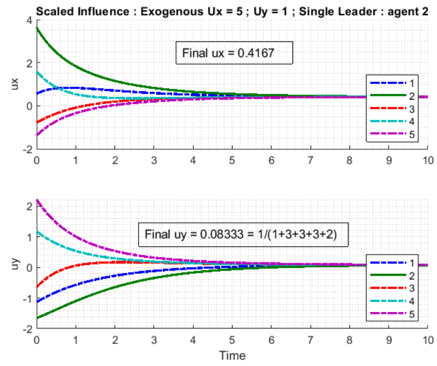

Figure 5: Velocities for Scaled Influence - Example 2.1

Example 2.2. *We let now the leader be agent 4 and show that for Scaled influence agents' speed is also a function of the individual selection of the leaders. Since its degree of agent 4 is 3, while the degree of agent 2 is 1, the speed in the scaled influence case triples compared to Example (2.1).*

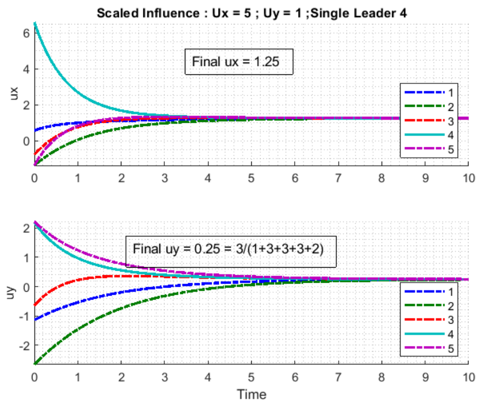

Figure 6: Velocities for System with Scaled Influences and agent 4 as leader

2.3.1 Time-varying inputs

Up to this point, we assumed time-invariant broadcast control and a time-invariant set of leaders. We next consider the impact of changes in these parameters.

- **Broadcast Control**: If the exogenous control direction or magnitude changes, the orientation or speed of the agents changes accordingly, thus trying to reach the asymptotic behavior corresponding to the current value and/or direction of the control.

  Fig. (7), (8) depict the behavior if a change in U happens at time $t = 7$.

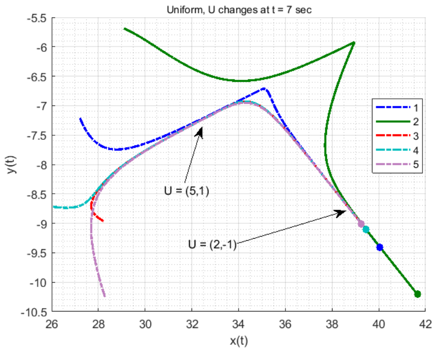

Figure 7: Trajectories with change in U at $t = 7$

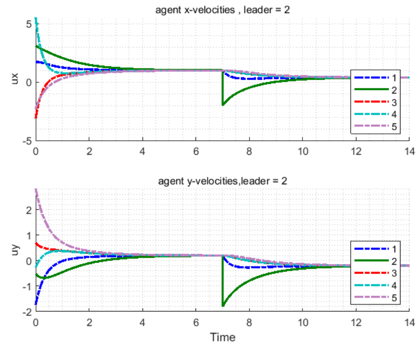

Figure 8: Velocities with change in U at $t = 7$

- **Set of leaders**: the emergent behavior depends on the number of leaders and possibly on their identity. Fig. (9) shows the change in agents' velocities caused by changes in the subset of leaders, while the exogenous control is kept fixed at $U = (5, 1)$.

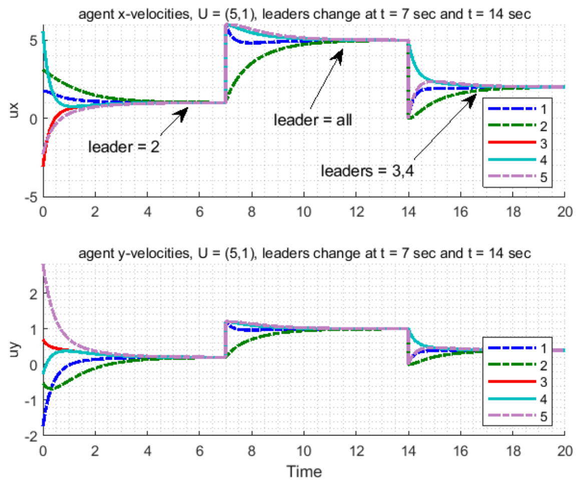

Figure 9: Velocities with change in leaders at $t = 7$ and $t = 14$

Note: the identity of the leaders impacts on the order and offsets of the agents along the direction of U, i.e. $\boldsymbol{\varrho}$, see Table (1).

3 Cyclic Pursuit

3.1 Introduction

The cyclic pursuit problem is formulated as n agents chasing each other. The agents are ordered from 1 to n, agent i acquires information about agent $(i \mod n) + 1$, referred to as *the successor* of agent i. For brevity, we suppress the modulo in the notation and replace index $(i \mod n) + 1$ with simply $i + 1$.

The cyclic pursuit can be represented by a directed cyclic graph, whose nodes are the agents and the directed edges depict the information flow.

In this work we consider two types of information acquired by the agents

1. Relative position of successor, leading to *Linear Cyclic Pursuit*
2. Bearing only information, i.e. direction to successor, leading to *Non-Linear Cyclic Pursuit*, known as the Bugs model

3.2 Linear Cyclic Pursuit with Broadcast Control

The asymptotic behavior in case of autonomous Linear Cyclic Pursuit has been discussed in the existing literature, see e.g. [4], [13]. Thus we concentrate on the case of Linear Cyclic Pursuit with Broadcast Control. In this model, the motion of agent i can be written as

$$\dot{p}_i(t) = p_{i+1}(t) - p_i(t) + b_i U \tag{6}$$

Eq. (6) can be decoupled and the evolution on each axis can be considered independently. Therefore we will treat only the dynamics on the x-axis. We have

$$\dot{x}(t) = Mx(t) + BU_x(t) \tag{7}$$

where M is the circulant matrix $M = circ[-1, 1, 0, 0, \dots, 0]$.

The solution of (7) is (cf. [12])

$$x(t) = e^{Mt}x(0) + \int_0^t e^{M(t-\tau)} BU_x \mathrm{d}\tau \tag{8}$$

We note that the Linear Cyclic pursuit is not a special case of the Linear dynamics with uniform influence, due to the directed cyclic topology.

We use the properties of the circulant matrix M, see Appendix 7.2 to obtain the following asymptotic position of agent i in the Linear Cyclic Pursuit Model, when an external control $U = (U_x \quad U_y)^T$ is detected by n_l agents, is

$$p_i(t \to \infty) = [\alpha + \beta U t + \varrho_i U] \tag{9}$$

where

- $\alpha = (\alpha_x \ \alpha_y)^T = \frac{1}{n}\sum_{i=1}^{n} p_i(0)$ is the agreement, or gathering, point when there is no external input
- $\beta = \dfrac{n_l}{n}$ and βU is the collective velocity.
- $\alpha + \beta U t$ is the position of the moving agreement point at time t
- $\varrho_i U$ is the offset of agent i from the moving agreement point, where

$$\boldsymbol{\varrho} = [\varrho_1, \varrho_2, \cdots, \varrho_n]^T = \left[\sum_{k=1}^{n-1} \frac{1}{\lambda_k} V_k V_k^*\right] B$$

with the following properties

1. $\sum_{i=1}^{n} \varrho_i = 0$
2. if $B = \mathbf{1}_n$, then $\varrho_i = 0; \quad \forall i = 1, \dots, n$

Here V^* represents the transposed complex conjugate of the vector V, namely $\overline{V}^T$, where $\overline{V}$ is the element by element complex conjugate of V.

3.2.1 Simulation results

We consider a system with 5 agents in Linear Cyclic Pursuit with initial configuration as in Fig. 10.

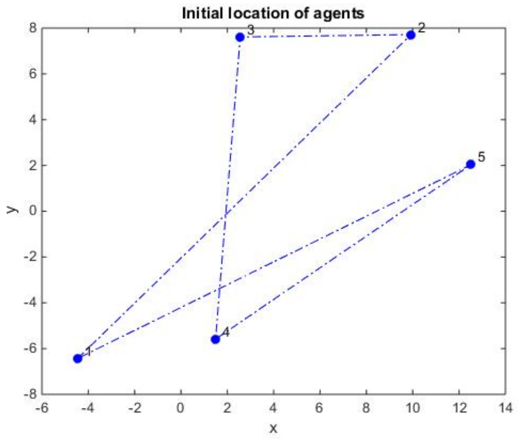

Figure 10: Cyclic Topology with Initial positions

Example 3.1. *Fig. 11, 12 show the behavior of the system with agent 2 as leader and broadcast control $U = (5, 1)$. We note that the asymptotic behavior in this case is similar to that of the linear model with uniform influence given in Example 2.1.*

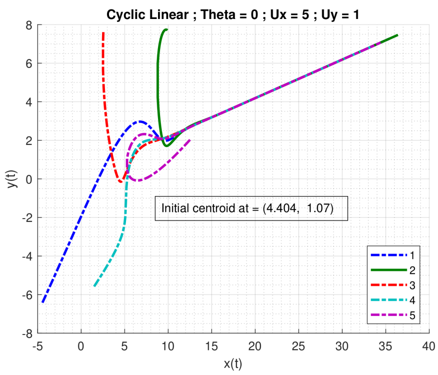

Figure 11: Trajectories for Example 3.1

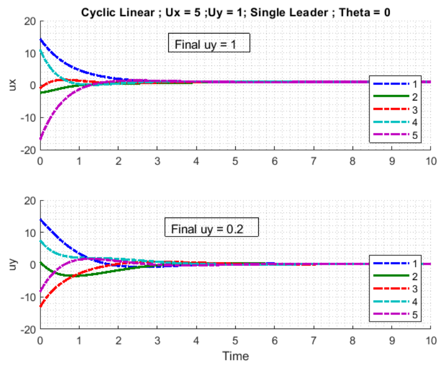

Figure 12: Velocities for Example 3.1

3.3 Bugs Model cyclic pursuit

In this section we model the autonomous dynamics of the agents according to the *Bugs* model. In the Bugs model agent i chases its successor, agent $i + 1$, *with unit speed* along the line of sight[3]. If agent i reaches agent $(i+1)$, we say that $(i+1)$ has *captured* i and the two combined agents continue with the dynamics of $i + 1$.

In the Bugs model, the dynamics is non-linear and the axes are not decoupled, thus new methods need to be employed in order to investigate the properties of the system.

Let $\delta_i(t)$ be the distance between agent i and its successor and $\Delta(t) = \sum_{1}^{n} \delta_i(t)$.

- **With no exogenous control**,
  1. $\delta_i(t)$ is monotonically non-increasing for all i.
  2. $\Delta(t)$ strictly decreases
  3. The bound on the *slope of descent* of $\Delta(t)$ strictly increases from interval to interval.
  4. All agents converge to a single point in finite time, see also [18].

[3] As before, the notation $i + 1$ is shorthand for $(i \mod n) + 1$.

- **With exogenous control**, if agent i is captured by its successor, then the combined agent is a leader if and only if either or both i and its successor have previously detected the exogenous control.

  We show that,

  – there exists a bound on the magnitude of the exogenous control $\|U\|$, that ensures that in finite time some agent captures all other agents. We note that this is only a *sufficient condition* for the bugs to converge to a single moving point, in the presence of an exogenous control.

  – the bound on $\|U\|$ is determined by the number of consecutive leader-follower and follower-leader pairs, denoted by n_b^0, in the initial configuration. A leader-follower or follower-leader pair is a pair of agents such that $|b_{i+1} - b_i| = 1$.

  ***Theorem* 3.1.** *If* $\|U\| < 1/(\sqrt{2}(n_b^0)^2)$*, then*

  1. *the bound on* $\Delta(t)$ *strictly decreases over the entire time*
  2. *the bound on the slope of descent of* $\Delta(t)$ *is non-decreasing from interval to interval*
  3. *all agents gather to a single moving point in finite time. The gathering point moves with velocity* U*.*

3.3.1 Simulation Results

Example 3.2. *Here we illustrate the behavior of the bugs model with exogenous broadcast control, starting from the initial positions shown in Fig. 10, with* $U = (0.5, 0.1)$ *and Leader=2.*

The agents converge to a common moving position, that moves with velocity U*.*

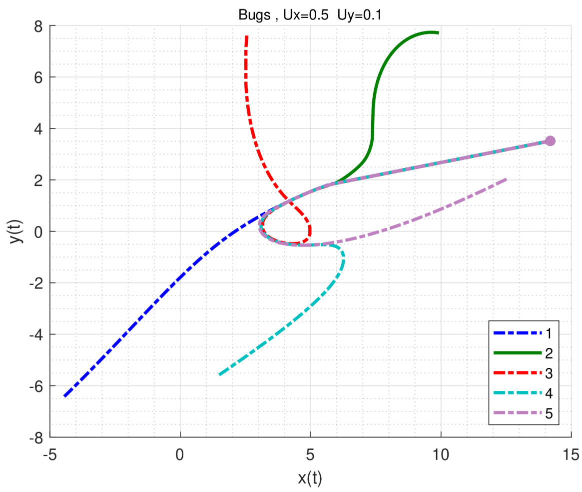

Figure 13: Bugs with Exogenous control U and one Leader

4 Deviated Cyclic Pursuit

The model treated in this section is the *Deviated cyclic pursuit*, where the pursuit line is rotated by a deviation angle θ from the line of sight to the successor. The deviation angle θ is assumed time independent and common to all agents. The positive direction of θ is taken to be *clockwise*.

4.1 Cyclic Deviated Linear dynamics with broadcast control

Pavone and Frazzoli, [14], introduced autonomous deviated cyclic pursuit, in the context of geometric planar pattern formations. We extend this model to deviated cyclic pursuit with a broadcast control, a velocity signal input.

In this case, the motion of agent i becomes

$$\dot{p}_i(t) = R(\theta)(p_{i+1}(t) - p_i(t)) + b_i U \tag{10}$$

where $R(\theta)$ is the rotation matrix and the collective movement of the swarm can be expressed as

$$\dot{P}(t) = \hat{M}P(t) + \hat{B}U \tag{11}$$

where $P(t)$ is the vector of stacked agent positions and $\hat{M}$ is the block circulant matrix with eigenvalues $\mu^{\pm}$, corresponding eigenvectors $\zeta^{\pm}$ and the properties shown in Appendix 7.3.

Eq. (11) is linear time independent and $\hat{M}$ is unitarily diagonizable, thus its solution can be written as

$$\begin{aligned} P(t) &= \frac{1}{2}\sum_{k=0}^{n-1}[\zeta_k^+ e^{\mu_k^+ t}(\zeta_k^+)^* + \zeta_k^- e^{\mu_k^- t}(\zeta_k^-)^*]P(0) \\ &+ \frac{1}{2}\int_0^t \sum_{k=0}^{n-1}\left[e^{\mu_k^+ \nu}\zeta_k^+(\zeta_k^+)^* + e^{\mu_k^- \nu}\zeta_k^-(\zeta_k^-)^*\right]\hat{B}U\mathrm{d}\nu \end{aligned} \tag{12}$$

We recall (see Appendix 7.3) that there exists a critical deviation angle $\theta_c = \dfrac{\pi}{n}$, such that

(a) if $|\theta| < \theta_c$, then $\hat{M}$ has two zero eigenvalues, $\mu_0^{\pm} = 0$, and all non-zero eigenvalues of $\hat{M}$ lie in the open left-half complex plane

(b) if $|\theta| = \theta_c$, then $\hat{M}$ has two zero eigenvalues, $\mu_0^{\pm} = 0$, and two non-zero eigenvalues which lie on the imaginary axis.

All remaining eigenvalues lie in the open left-half complex plane.

(c) if $|\theta| > \theta_c$, then $\hat{M}$ has two zero eigenvalues and at least two non-zero eigenvalues which lie in the open right-half complex plane.

We consider two cases, $|\theta| < \theta_c = \pi/n$ and $|\theta| = \theta_c = \pi/n$ (the case $|\theta| > \theta_c = \pi/n$ renders an unstable system and will not be discussed here).

The asymptotic behavior in each case is obtained by decomposing Eq. (12) into terms containing eigenvalues with zero real part and terms containing eigenvalues with negative real part and letting $t \to \infty$.

4.2 Asymptotic behavior when $|\theta| < \theta_c = \pi/n$

For $|\theta| < \theta_c$, as $t \to \infty$, we have

$$\begin{aligned} P(t\to\infty) &= \frac{1}{2}[\zeta_0^+(\zeta_0^+)^* + \zeta_0^-(\zeta_0^-)^*]P(0) \\ &+ \frac{1}{2}\left[(\zeta_0^+(\zeta_0^+)^* + \zeta_0^-(\zeta_0^-)^*)\right]\hat{B}Ut \\ &+ \frac{1}{2}\sum_{k=1}^{n-1}\left[\frac{1}{\mu_k^+}\zeta_k^+(\zeta_k^+)^* + \frac{1}{\mu_k^-}\zeta_k^-(\zeta_k^-)^*\right]\hat{B}U \end{aligned} \tag{13}$$

Calculating the various parts of Eq. (13), we obtain

Theorem 4.1. *The asymptotic position of agent i in the Deviated Linear Pursuit Model with deviation angle θ, such that $|\theta| < \theta_c$, when an external control $U = (U_x \quad U_y)^T$ is detected by a subset of n_l agents, is*

$$p_i(t\to\infty) = [\alpha + \beta Ut + \varrho_i R(-\theta)U] \tag{14}$$

where

- *α is the centroid of initial positions*
- $\beta = \dfrac{n_l}{n}$
- *the vector of offsets is rotated by $-\theta$ from the direction of U, around the common moving point.*

We note that the only impact of the deviation angle $0 < \theta < \theta_c$ is on the asymptotic alignment of the agents. This is illustrated in Example 4.1

Example 4.1. *Here we show the behavior of a system of 5 agents, thus $\theta_c = 36$ deg, with Broadcast Control $U = (5, 1)^T$ with agent 2 as leader and deviation angle $\theta = 20\,deg < \theta_c$. Fig. 14 and 15 depict the agents' trajectories and velocities. The velocity of all agents asymptotically converges to $\frac{n_l}{n}U = 0.2U$, where $n_l = 1$ is the number of leaders. The asymptotic trajectories are parallel lines in the direction of U. The agents' offsets for both $\theta = 0$ and $\theta = 20\,deg$ are depicted in Fig. 16. When $\theta = 0$, the offsets are aligned with slope 0.2 along the direction of U, while with $\theta = 20\,deg$, we have $\cos\theta = 0.94$ and $\sin\theta = 0.34$, thus $R(-\theta)U$ has a slope= 0.608.*

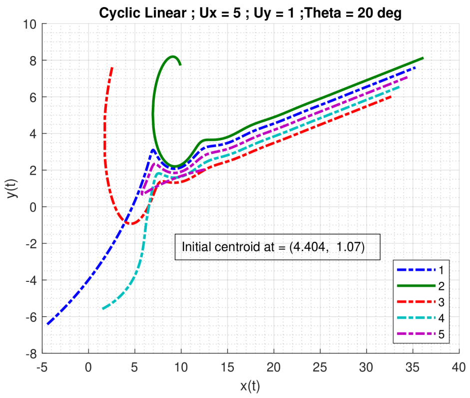

Figure 14: $\theta = 20\,deg$ - Agents' Trajectories

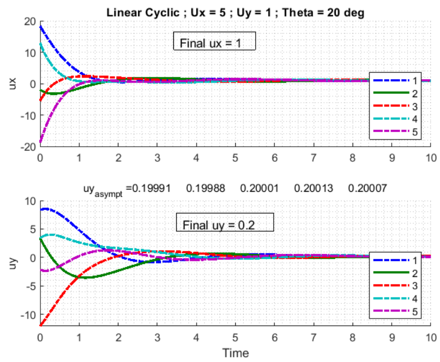

Figure 15: $\theta = 20$ deg - Agents' Velocities

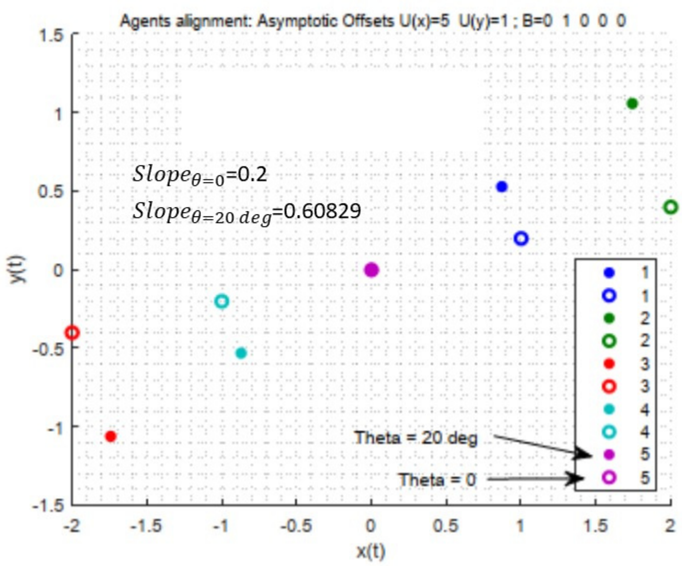

Figure 16: $\theta = 20\,deg$ vs. $\theta = 0\,deg$ - Offsets

4.3 Asymptotic behavior when $|\theta| = \theta_c = \pi/n$

If $|\theta| = \theta_c$, then $\hat{M}$ has *two additional eigenvalues* with zero real part.

Thus, in $P(t \to \infty)$ we have additional terms, compared to the case $|\theta| < \theta_c$. The additional terms are

$$\frac{1}{2}\left[e^{\mu^+_{n-1}t}\zeta^+_{n-1}(\zeta^+_{n-1})^*[P(0) - \frac{1}{\mu^+_{(n-1)}}\hat{B}U] + e^{\mu^-_1 t}\zeta^-_1(\zeta^-_1)^*[P(0) - \frac{1}{\mu^-_1}\hat{B}U]\right]$$

After some algebra we obtain the results summarized by Theorem 4.2

Theorem 4.2. *When the deviation angle is at the critical value, i.e. $\theta = \theta_c$, and an exogenous velocity signal $U = (U_x \quad U_y)^T$ is detected by n_l leaders, then asymptotically*

1. *Each agent moves in steady state in a circle around a moving center.*
   - *The radius is determined by the initial positions and the subset of leaders*
   - *When all agents are leaders the radius is determined solely by the initial positions*
   - *The center of the circular movement of agent i moves asymptotically on the line*

   $$p_i(t \to \infty) = [\alpha + \beta U t + \varrho_i R(-\theta_c)U] \tag{15}$$

   where α, β, ϱ_i are as in Theorem 4.1
2. *All circles have the same radius and the rotation is with the same angular velocity.*
3. *The agents' phases are equally angular distanced.*

4.3.1 Illustration

Example 4.2. *Here we show the behavior of a system with Broadcast Control $U = (5, 1)^T$ with agent 2 as leader and deviation angle $\theta = 36\,deg = \theta_c$. Figs. 17 and 18 depict the agents' trajectories and velocities. The agents move asymptotically in circles along a line with slope 0.2.*

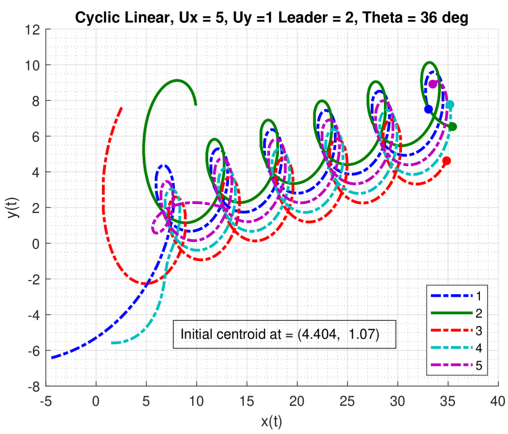

Figure 17: $\theta = 36\,deg$ - Agents' Trajectories

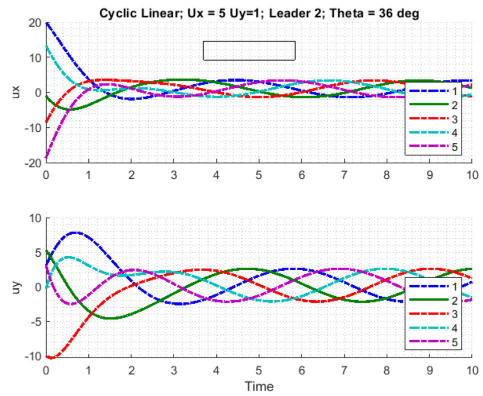

Figure 18: $\theta = 36\,deg$ - Agents' Velocities

4.4 Deviated Bugs with broadcast control

In the ***Deviated Bugs*** **Model** the autonomous speed of the agents is 1 and the direction is deviated by θ from the line of sight to the successor. The leaders add to this movement the detected velocity. The capturing mechanism and properties are identical to the Non-deviated Bugs model. As such only the non-captured agents at any time are relevant.

- With **no exogenous control**, we show that

  - if the number of non-captured agents is or becomes strictly less than π/θ, then in finite time all agents except one are captured by the remaining agent.
  - if the number of non-captured agents is or becomes equal to π/θ, then the non-captured agents' positions converge to a regular polygon and move with unit linear velocity on the circumscribed circle

- With **exogenous control**
  - if $\theta < \theta_c$ then
    * if $\theta < \arccos(1 - \frac{1}{\sqrt{2}nn_b^0})$, where n_b^0 is the initial number of leader-follower or follower-leader pairs, then there exists a bound on $\|U\|$ that ensures convergence to a single moving point.
    * Simulations show that the convergence property is not limited by the above bounds
    * The convergence point moves with velocity U
  - if $\theta = \theta_c$ then
    * an analytical derivation of agents' behavior eludes us
    * simulations were non-conclusive

5 Systems with finite visibility

In this section we treat systems with finite visibility. We first consider piecewise linear systems with Uniform influence and time invariant parameters, where links between agents may connect or disconnect when they approach the visibility range. Moreover, the graph may disconnect or re-connect. This model will be referred to in the sequel simply as a piecewise linear system.

Then we introduce and treat Potential Based systems. These are non-linear models with the property that without exogenous input, the links between agents never disconnect.

5.1 Systems with Piecewise Linear Dynamics and Finite Visibility

In this section we investigate the behavior of a system of agents with finite visibility, piecewise linear dynamics and Uniform influence. The dynamics of the system between topological changes caused by finite visibility is as in Section 2. The system dynamics is

$$\dot{p}_i = \sum_{j \in N_i} (p_j - p_i) + b_i U \tag{16}$$

where $j \in N_i$ **if and only if** $\|p_i - p_j\| \leq R$. We distinguish between several types of events:

1. Link disconnection: $\|p_i - p_j\|$ becomes $> R$ for some $j \in N_i$
2. Link Connection: $\|p_i - p_j\|$ becomes $\leq R$ for some $j \notin N_i$
3. Graph disconnection: graph decomposition into disconnected subgraphs due to link disconnection
4. Subgraph connection: Connection of a link between two nodes belonging to two previously disconnected subgraphs.
5. Topological change: any of the above

We show that

- even without exogenous control the graph may disconnect and in fact there is no exogenous control or visibility range that ensures that the agents graph never disconnects
- assuming that the agents' visibility graph is initially connected

– If the graph remains connected for a sufficiently long time, then the agents positions converge exponentially to a single line parallel to the direction of U and anchored at the centroid of the agents' initial positions
– Otherwise, allowing for topological changes that give rise to graph disconnection and reconnection, agents' positions converge exponentially to one or more line(s) parallel to the direction of U, corresponding to the sub-graphs that have resulted from the topological changes.

5.2 Potential Based Dynamics

We consider the Potential Based Model shown to ensure, in autonomous systems, links preservation ("never lose neighbors") and investigate this property when an exogenous control, broadcast to the swarm, is detected by a subset of agents.

We show that there is no fixed exogenous input U that preserves this property, i.e. ensures links preservation. In order to solve this problem, we modified our model such that the leaders employ an **adaptive gain** for the detected control. The gain is a function of leader's distance from its neighbors, such that it vanishes whenever any neighbor approaches the visibility range R. We show that with this adaptive gain on the exogenous input the property of never losing neighbors is preserved.

5.2.1 Definition and properties of distributed potential functions

Consider a set of distributed smooth functions, referred to as *potential functions,* of the form $\varphi_i(\sigma_i, \pi_i)$ with the properties

$$\frac{\partial \varphi_i}{\partial \sigma_i}(\sigma_i, 0) = 0\,; \;\; \frac{\partial \varphi_i}{\partial \pi_i}(\sigma_i, 0) < 0 \text{ for } \sigma_i \geq 0, \tag{17}$$

where

$$\sigma_i(t) = \frac{1}{2R^2} \sum_{j \in N_i(t)} \|p_i(t) - p_j(t)\|^2 \tag{18}$$

$$\pi_i(t) = \frac{1}{2} \prod_{j \in N_i(t)} \left(1 - \frac{\|p_i(t) - p_j(t)\|^2}{R^2}\right) \tag{19}$$

***Theorem* 5.1.** *[1] The autonomous dynamics of the agents*

$$\dot{p}_i = -\frac{\partial \varphi_i}{\partial p_i} \tag{20}$$

is neighbors-preserving, namely for all pairs i, j for which $j \in N_i(t_0)$ holds $j \in N_i(t)$; for all $t \geq t_0$.

5.2.2 Definition and properties of distributed navigation-like potential functions

The navigation-like potential function, referred to as the NAV function, is defined by

$$\varphi_i(\sigma_i, \pi_i) = \frac{\sigma_i}{\sigma_i + \pi_i} \tag{21}$$

(cf. [3], [5],[6]) . This potential function complies with the conditions (17), therefore it is connectivity preserving. In addition it also leads to gathering of the agents at a common position.

5.3 Emergent behavior with the NAV potential and exogenous control

In this system the leaders apply variable gains on the detected exogenous control. We assume the gains to be π_i; $i \in N^l$, shown to be connectivity preserving.

The dynamics of the system is:

$$\dot{p}_i = -\sum_{j \in N_i} a_{ij}(p_i - p_j) + b_i \pi_i U \tag{22}$$

with a_{ij} defined by

$$a_{ij} = \frac{\pi_i + \sigma_i \pi_{ij}}{R^2(\sigma_i + \pi_i)^2} > 0 \tag{23}$$

where π_{ij} is the same as π_i without considering agent j.

***Theorem* 5.2.** *With system dynamics (22), the agents positions align asymptotically along a line in the direction of the exogenous control U.*

Asymptotically agents' dynamics has an equilibrium state, where all agents move with the same velocity.

***Theorem* 5.3.**

With system dynamics (22), the system has a stable equilibrium state with the following properties:

1. *the leaders have a common moving position $p^{(l)}(t)$*
2. *the followers have a common moving position $p^{(f)}(t)$*
3. *all agents move with the same speed $\|\dot{p}^{(l)}\| = \|\dot{p}^{(f)}\|$ in the direction of U*
4. *the leaders-followers normalized distance $\hat{\delta} = \dfrac{\|p^{(l)} - p^{(f)}\|}{R}$ and their speed obey the non-linear equation*

$$\dot{p}^{(l)} = \dot{p}^{(f)} \tag{24}$$

$$\begin{aligned} \dot{p}^{(l)} &= -\frac{n_f a^{(lf)} \hat{\delta} R}{\|U\|} U + \pi^{(l)} U \\ \dot{p}^{(f)} &= \frac{n_l a^{(fl)} \hat{\delta} R}{\|U\|} U \end{aligned} \tag{25}$$

where n_l, n_f is the number of leaders and followers respectively. with

$$\begin{aligned} \sigma^{(l)} &= \frac{n_f}{2}\hat{\delta}^2 \; ; & \sigma^{(f)} &= \frac{n_l}{2}\hat{\delta}^2 \\ \pi^{(l)} &= \frac{1}{2}\left(1 - \hat{\delta}^2\right)^{n_f} \; ; & \pi^{(f)} &= \tfrac{1}{2}\left(1 - \hat{\delta}^2\right)^{n_l} \\ \pi^{(lf)} &= \left(1 - \hat{\delta}^2\right)^{n_f - 1} \; ; & \pi^{(fl)} &= \left(1 - \hat{\delta}^2\right)^{n_l - 1} \\ a^{(lf)} &= \frac{\pi^{(l)} + \sigma^{(l)}\pi^{(lf)}}{R^2(\sigma^{(l)} + \pi^{(l)})^2} \; ; & a^{(fl)} &= \frac{\pi^{(f)} + \sigma^{(f)}\pi^{(fl)}}{R^2(\sigma^{(f)} + \pi^{(f)})^2} \end{aligned} \tag{26}$$

6 Concluding Remarks

We have investigated the behavior of a system of agents moving in the plane, in the presence of a broadcast guidance control, a velocity signal, detected by a subset of agents, referred to as leaders. The number of leaders and their identity are arbitrary. The autonomous dynamics is common to all agents. Leaders add the detected velocity to their autonomous velocity, possibly moderated by adaptive gains (in Potential Based Dynamics). This is an innovative paradigm applied to various models of autonomous dynamics, linear and non-linear, fixed as well as position dependent topologies.

For each of these models we investigate the effect of the broadcast velocity and of the subset of leaders on the emergent behavior of the system.

For Bearing-only cyclic pursuit we obtained only bounds on the magnitude of the broadcast velocity that ensure convergence. These bounds are only sufficient and not necessary conditions, as illustrated by simulations.

Future work should mostly concentrate on further analysis of the Potential Based Model. In our work we have shown that there can be one or more equilibrium states that are solutions of an induced non-linear equation and for small values of the parameters, the system converges to an equilibrium state. All numerical solutions and simulations show that the equilibrium point is unique. Further research is still necessary to rigorously prove these properties.

Detailed proofs of the results surveyed in this paper, as well as extensive simulations, of swarm systems with broadcast control may be found in ([19], ([20], ([21], ([22], ([23].

7 Appendices

7.1 Laplacian for the Uniform and Scaled cases

The Laplacian L is defined as

$$L_{ij} = \begin{cases} -\sigma_{ji} & \text{if } j \in N_i \text{ and } j \neq i \\ \sum_{j\neq i} \sigma_{ji} & \text{if } j = i \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

In both cases

- All eigenvalues of L, denoted by λ_i, are real and non-negative
- There is a single zero eigenvalue, λ_0, and all remaining eigenvalues are strictly positive
- V_0, the eigenvector corresponding to $\lambda_0 = 0$, is $\frac{1}{\sqrt{n}}\mathbf{1}_n$
- L is diagonizable, thus can be written as

$$L = V\Lambda W^T$$

  where V is the matrix of right eigenvectors, Λ is the diagonal matrix of eigenvalues and $W^T = V^{-1}$, for the Uniform or the Scaled case. We note that for the Scaled case

  - Each row of $(W)^T$ is a left eigenvector of L
  - The first row of $(W)^T$, denoted by $(W)^T_0$, is a left eigenvector of L corresponding to $\lambda_0 = 0$ and we have

$$(W)^T_0 = \frac{\sqrt{n}\cdot \mathbf{d}^T}{\sum_{i=1}^n d_i} \tag{28}$$

    where $\mathbf{d}$ is the vector of degrees in the connectivity graph.

7.2 Properties of a circulant matrix M

M is unitarily diagonizable, i.e. $M = V\Lambda V^*$, where V is a unitary matrix of eigenvectors, V^* denotes the transpose of the complex conjugate of V and Λ is a diagonal matrix of eigenvalues.

- the eigenvalues of M are

$$\lambda_k = e^{-2\pi jk/n} - 1; \quad k = 0, ..., n-1 \tag{29}$$

  where $j = \sqrt{-1}$
- with corresponding eigenvectors

$$V_k = \frac{1}{\sqrt{n}}\left(1, e^{-2\pi jk/n}, e^{-4\pi jk/n}, \ldots, e^{-(n-1)\pi jk/n}\right)^T; \quad k \in \{0, 1, \ldots, n-1\} \tag{30}$$

The first eigenvalue is $\lambda_0 = 0$, with the corresponding eigenvector $V_0 = \frac{1}{\sqrt{n}}\mathbf{1}_n$, while the remaining eigenvalues, λ_k; $k = 1, ..., n-1$, have a negative real part.

7.3 Properties of a block circulant matrix $\hat{M}$

- $\hat{M} = M \otimes R(\theta)$, where $M = circ[-1, 1, 0, 0, \ldots, 0]$ and $R(\theta)$ is the rotation matrix. Therefore, $\hat{M}$ is block circulant, as follows:

$$\hat{M} = \begin{bmatrix} -R(\theta) & R(\theta) & 0_{2\times 2} & 0_{2\times 2} & \ldots & 0_{2\times 2} \\ 0_{2\times 2} & -R(\theta) & R(\theta) & 0_{2\times 2} & \ldots & 0_{2\times 2} \\ & & \vdots & & & \\ R(\theta) & 0_{2\times 2} & \ldots & & 0_{2\times 2} & -R(\theta) \end{bmatrix} \tag{31}$$

- $\hat{B} = B \otimes I$, where B is the leaders vector and I is the 2×2 identity matrix
- $\otimes$ denotes the Kronecker product
- U is the external broadcast control, $U \in \mathbb{R}^2$

$\hat{M}$ is unitarily diagonizable as

$$\hat{M} = \hat{V}\hat{\Lambda}\hat{V}^* \tag{32}$$

where

- $\hat{\Lambda}$ is the diagonal matrix of eigenvalues of $\hat{M}$
- $\hat{V}$ is a unitary matrix of the corresponding eigenvectors of $\hat{M}$ and $\hat{V}^*$ denotes the transpose of the complex conjugate of $\hat{V}$

The eigenvalues of $\hat{M}$ are $\mu_k^{\pm}; \quad k = 0, \ldots, n-1$, given by

$$\mu_k^{\pm} = \lambda_k e^{\pm j\theta} = 2\sin(\frac{\pi k}{n})e^{-j(\frac{\pi}{2}+\frac{\pi k}{n}\pm\theta)} \tag{33}$$

The corresponding eigenvectors, denoted by $\zeta_k^{\pm}$, are given by $\zeta_k^{\pm} = V_k \otimes (1, \pm j)^T$, where

$$V_k = \frac{1}{\sqrt{n}}\left(1, e^{-2\pi kj/n}, e^{-4\pi kj/n}, \ldots, e^{-2\pi kj(n-1)/n}\right)^T; k = (0, ..., n-1) \tag{34}$$

We define the critical deviation angle $\theta_c = \dfrac{\pi}{n}$.
Then, we have

(a) if $|\theta| < \theta_c$, then $\hat{M}$ has two zero eigenvalues, $\mu_0^{\pm} = 0$, and all non-zero eigenvalues of $\hat{M}$ lie in the open left-half complex plane

(b) if $|\theta| = \theta_c$, then $\hat{M}$ has two zero eigenvalues, $\mu_0^{\pm} = 0$, and two non-zero eigenvalues which lie on the imaginary axis

   - $\mu_1^{-} = -2j\sin(\frac{\pi}{n})$
   - $\mu_{n-1}^{+} = 2j\sin(\frac{\pi}{n})$

   All remaining eigenvalues lie in the open left-half complex plane.

(c) if $|\theta| > \theta_c$, then $\hat{M}$ has two zero eigenvalues and at least two non-zero eigenvalues which lie in the open right-half complex plane.

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