

Search for Smart Evaders With Swarms of Sweeping Agents

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Abstract—Suppose in a given planar region, there are smart mobile evaders and we want to detect them using sweeping agents. We assume that the agents have line sensors of equal length. We propose procedures for designing cooperative sweeping processes that ensure successful completion of the task, thereby deriving conditions on the sweeping velocity of the agents and their paths. Successful completion of the task means that evaders with a known limit on their velocity cannot escape detection by the sweeping agents. A simpler task for the sweeping swarm is the confinement of the evaders to their initial domain. The feasibility of completing these tasks depends on geometric and dynamic constraints that impose a lower bound on the velocity the sweeping agent must have. This critical velocity is derived to ensure the achievement of the confinement task. Increasing the velocity above the lower bound enables the agents to complete the search task as well. We present results on the total search time for two types of novel pincer-movement search processes, circular and spiral, for any even number of sweeping agents. The proposed spiral process allows detection of all evaders while sweeping at velocities that approach the theoretical lower bound.

Index Terms—Aerial robots, cooperating robots, motion planning, multiple mobile robot systems, robot surveillance and security, search and rescue robotics, swarms.

I. INTRODUCTION

THE aim of this work is to provide an efficient “must-win” search policy for a swarm of n sweeping agents that must guarantee detection of an unknown number of smart evaders initially residing inside a given circular region of radius R_0 while minimizing the search time. The evaders move and try to escape the initial region at a maximal velocity of V_T , known to the sweepers. All sweepers move at a velocity $V_s > V_T$ and detect the evaders using linear sensors of length $2r$. Each “must-win” policy requires a minimal velocity that depends on the trajectory

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This article has supplementary downloadable material available at <http://ieeexplore.ieee.org>, provided by the authors. The material consists of a video, viewable with QuickTime or Windows Media Player, which presents the objectives of this article and demonstrates using illustrative simulations the evolution of the search processes described in this article, and also presents the key results of this article. The size of the video is 52.3 MB. Contact roe.francos@cs.technion.ac.il for further questions about this work.

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of the sweepers. Finding an efficient algorithm requires that, throughout the sweep, the footprint of the sweepers’ sensors maximally overlaps the evader region (the region where evaders may possibly be). This work develops two “must-win” search strategies for a swarm consisting of an even number of searchers that sweep the evader region until all evaders are detected, by employing some novel pincer movement search strategies. The search is based on pairs of agents sweeping toward each other thereby entrapping all evaders.

A. Overview of Related Research

An interesting challenge for multiagent systems is the design of searching or sweeping algorithms for static or mobile targets in a region, which can either be fully mapped in advance or unknown, see, e.g., [1]–[4]. Often the aim is to continuously patrol a domain in order to detect intruders or to systematically search for mobile targets known to be located within a given area [5]. Search for static targets involves complete covering of the area where they are located, but a much more interesting and realistic scenario is the question of how to efficiently search for targets that are dynamic and smart. A smart target is one that detects and responds to the motions of searchers by performing optimal evasive maneuvers, to avoid interception. A smart target is assumed to have full knowledge of the searching swarm’s patrol strategy. Therefore, it can utilize this knowledge in order to plan its movements in a way that will maximize the time it takes the pursuing sweepers to detect it. Guaranteed detection of all evaders means that for all particular choices of trajectories, a smart target may implement in order to escape the pursuing agents, it will surely be eventually detected.

Several such problems originated in the second world war due to the need to design patrol strategies for aircraft aiming to detect ships or submarines in the English channel, see [6]. The problem of patrolling a corridor using multiagent sweeping systems in order to ensure the detection and interception of smart targets was also investigated by Vincent and Rubin in [7] and provably optimal strategies were provided by Altshuler *et al.* [8]. A somewhat related, discrete version of the problem, was also investigated by Wagner and Bruckstein and later by Altshuler *et al.* [9]–[11]. It focuses on a dynamic variant of the cooperative cleaners’ problem, a problem that requires several simple agents to a clean and connected region on the grid with contaminated pixels. This contamination is assumed to spread to neighbors at a given rate.

Bressan *et al.* [12]–[15] investigate optimal strategies for the construction of barriers in real time aiming at containing and confining the spread of fire from a given initial area of the plane. The authors are interested in determining a minimal possible barrier construction speed that enables the confinement of the fire, and on determining optimality conditions for confinement strategies. Bressan *et al.* [12] define the barrier curve construction as an optimization problem by introducing a cost functional that takes into account the size of the area destroyed by fire in addition to the total cost of building the barrier walls. The goal is to fully enclose the fire in finite time by the walls, thereby stopping the fire's spread. Bressan *et al.* [12] propose necessary and sufficient conditions for the construction of an optimal barrier, suggesting that an optimal strategy for confining the spread of fire from an initial circular area in the plane should be the building of logarithmic spiral firewalls that track the fire's wavefront as well as building a delaying arc, whose purpose is to delay the spreading of the fire from a particular direction, before enclosing it completely. An additional work that builds a barrier against an advancing fire using a spiral out pattern is given in [16] by Klein *et al.* [16] The construction of logarithmic spiral barriers is performed along the boundary of the expanding fire, carried out by a fire fighter with a pointlike "sensor." Similarly to the works of Bressan *et al.* [12], the building of the barrier is successful when the barrier curve closes, thus containing the fire within. Klein *et al.* [16] provide a proof of a lower bound on a fire fighter's velocity needed in order to construct a barrier that contains the spread of fire from an arbitrary sized circular region of the plane. Interestingly, the lower bound for the ratio between the firefighter's and fire's velocities equals the golden ratio.

Tang and Ozguner [5] propose a nonescape search procedure for evaders. Evaders are originally located in a convex region of the plane and may move out of it. Tang and Ozguner [5] propose a cooperative progressing spiral-in algorithm performed by several agents with disk-shaped sensors in a leader–follower formation. The authors establish a sufficient condition for the number of searching agents required to guarantee that no evader can escape the region undetected. This lower bound is based on the sensor radius, searcher and evader velocities, and the initial perimeter of the region. McGee and Hedrick [17] also investigate a search problem for smart targets that do not have any maneuverability restrictions except for an upper limit on their velocity. The sensor that the agents are equipped with detects targets within a disk-shaped area around the searcher location. Search patterns consisting of spiral and linear sections are considered. In [18], Hew proposes searching for smart evaders using concentric arc trajectories with agents' sensors similar to McGee and Hedrick [17]. Such a search is aimed at detecting submarines in a channel or in a half-plane. This article focuses on determining the size of a region that can be successfully patrolled by a single searcher, where the searcher and evader velocities are known. The search problem is formulated as an optimization problem. The search progress per arc or linear iteration is maximized while guaranteeing that the evader cannot slip past the searcher undetected.

Another set of related problems are pursuit-evasion games, where the pursuers' objective is to detect evaders and the

evaders' objective is to avoid the pursuers. Pursuit-evasion games include combinations of single and multiple evaders and pursuers scenarios. In this context, several works considered the problem of defending a region from the entrance of intruders. In [19]–[21], such problems are investigated under the name of "reach-avoid games." These types of problems were also addressed in the context of perimeter defense games by Shishika *et al.* [22]–[24], with a focus on utilizing cooperation between pursuers to improve the defense strategy. In [22], implicit cooperation between pairs of defenders that move in a "pincer movement" is performed in order to intercept intruders before they enter a convex region in the plane. This cooperation extends previous two player intruder–defender differential games that assign one intruder per defender and proposes a decomposition of the defense problem into local subproblems. The authors show that the cooperation among defender subteams enlarges the winning areas of the defenders. In [24], the authors extend their previous results from Shishika and Kumar [22] and propose an improved polynomial-time algorithm for the perimeter defense problem as well as addressing the optimality of the strategies by deriving a condition for which the lower bound on the game score of the intruder team equals the upper bound of the defender team. Agmon *et al.* [25]–[27] develop multirobot perimeter patrol strategies under adversarial settings against an opponent with full knowledge of the patrol. Since a smart opponent knows the robots' patrol strategy, it can utilize this knowledge in order to penetrate the perimeter at a point that maximizes its probability to enter the guarded area undetected. Therefore, the authors propose that robots perform nondeterministic patrol algorithms around the perimeter of a given region in order to maximize the probability of detecting the opponent. Guaranteeing defense of a region from entrance of intruders can be considered as a dual problem of detecting all evaders in a known region without allowing for any evader to escape the region undetected.

In our previous work [28], the confinement and cleaning tasks for a line formation of agents or alternatively for a single agent with a linear sensor are analyzed. Several methods are proposed on how to determine the minimal velocity for a circularly sweeping agent, in order to shrink the evader region within a circle with a smaller radius than half the searcher's sensor length. The results show that this velocity equals more than twice the theoretical lower bound. Furthermore, a proof that a single agent or a line formation of agents employing a circular search around the evader region cannot completely clean the evader region without modifying the search pattern is provided. Finally, this article describes a modification to the trajectory of the sweepers at the final sweep around the region that allows to clean it from all evaders.

B. Contributions

We present a comprehensive theoretical and numerical analysis of trajectories, critical velocities, and search times for a swarm of n cooperative agents whose mission is to guarantee detection of all smart evaders that are initially located in given circular region from which they may move out of in order to escape the pursuing sweeping agents.

- 1) We present following two types of novel search strategies:
 - a) n -agent circular pincer sweep strategy;
 - b) n -agent spiral pincer sweep strategy.
- 2) We develop analytic formulas for the two types of search patterns, for any even number of sweeping agents employing the pincer search protocols.
- 3) Circular pincer sweep process: We prove that sweeping with pairs of sweepers employing pincer movements between themselves and between adjacent sweeper pairs yields a lower critical velocity than the case where sweepers are distributed evenly around the region and sweep in the same direction.
- 4) Spiral pincer sweep process: We provide an algorithm that guarantees successful detection of all evaders in the region while sweeping with velocities that approach the theoretical lower bound on the velocity.
- 5) We compare circular and spiral pincer sweep strategies, showing the superiority of the latter.
- 6) We provide a numerical quantitative study demonstrating the performance advantages of pincer-based strategies over their same-direction counterparts.
- 7) The theoretical analysis is complemented by simulation experiments in MATLAB and NetLogo that verify the theoretical results and illustrate them graphically.

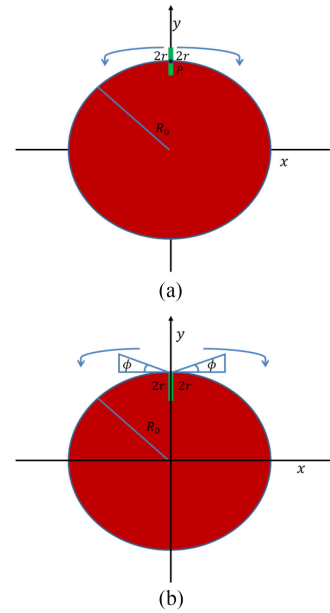


Fig. 1. (a) Initial placement of two agents employing the circular sweep process. (b) Initial placement of two agents employing the spiral sweep process. The sweepers' sensors are shown in green. The angle ϕ is the angle between the tip of a sweeper's sensor and the normal of the evader region. ϕ is an angle that depends on the ratio between the sweeper and evader velocities.

C. Paper Organization

This article is organized as follows. Section II describes the motivation and setting for using pincer-based search strategies. Section III proves an optimal bound on the cleaning rate for a swarm that is independent of the search process that is deployed. This bound will serve as one of the benchmarks for comparing the performance of different search algorithms. In Section IV, the results for the completion of the search process for a swarm of sweeping agents that employ the circular pincer sweep process are presented. In Section V, we perform an analysis for the case where the swarm employs the spiral pincer sweep process. In Section VI, we provide a comparative unified analysis of the proposed search strategies that were developed in the previous sections. In Section VII, we provide a quantitative comparison between pincer-based and same-direction search strategies. Finally, Section VIII concludes this article.

II. PINCER-BASED SEARCH

This article considers a scenario in which a multiagent swarm of identical agents search for mobile targets or evaders that are to be detected. The information the agents perceive only comes from their own sensors, and all evaders that intersect a sweeper's field of view are detected. We assume that all agents have a linear sensor of length $2r$. The evaders are initially located in a disk-shaped region of radius R_0 . There can be many evaders, and we consider the domain to be continuous, meaning that evaders can be located at any point in the interior of the circular region at the beginning of the search process. The sweeping protocols proposed are predetermined and deterministic, hence the sweepers can perform them using a minimal amount of memory and computations. All sweepers move with a speed of V_s (measured

at the center of the linear sensor). By assumption, the evaders move at a maximal speed of V_T , without any maneuverability restrictions. The sweeper swarm's objective is to "clean" or to detect all evaders that can move freely in all directions from their initial locations in the circular region of radius R_0 .

Search time clearly depends on the type of sweeping movement the swarm employs. Detection of evaders is based on deterministic and preprogrammed search protocols. We consider two types of search patterns, circular and spiral patterns. The desired result is that after each sweep around the region, the radius of the circle that bounds the evader region (for the circular sweep), or the actual radius of the evader region (for the spiral sweep), will decrease by a value that is strictly positive. This guarantees complete cleaning of the evader region, by shrinking in finite time the possible area in which evaders can reside to zero. At the beginning of the circular search process, we assume that only half the length of the agents' sensors is inside the evader region, i.e., a footprint of length r , whereas the other half is outside the region in order to catch evaders that may move outside the region while the search progresses. At the beginning of the spiral search process, we assume that the entire length of the agents' sensors is inside the evader region, i.e., a footprint of length $2r$.

In the single-agent search problem described in [28], we observed that there can be escape from point $P = (0, R_0)$ [see Fig. 1(a)], when basing the searcher's velocity only on a single traversal around the evader region. Therefore, we had to increase the agent's critical velocity to deal with this possible escape. If we were to distribute a multiagent swarm say, equally along the boundary of the initial evader region, we would have the same problem of possible escape from the points adjacent to the

starting locations of the sweepers. Since we wish the sweepers to have the lowest possible critical velocity, we propose a different idea for the search process. The idea is to have pairs of sweeping agents move out in opposite directions along the boundary of the evader region and sweep in a pincer movement rather than having a convoy of sweepers all moving in the same direction along the boundary.

Our method is readily applicable for any even number of sweepers. The sweepers are initially positioned in pairs back to back. One sweeper in the pair moves counter clockwise while the other sweeper in the pair moves clockwise. In case the search is planar, once the sweepers meet, i.e., their sensors are again superimposed at a meeting point, they switch the directions in which they move. For example, if the search is carried out by two sweepers, after the first sweep, this switching point is located at $(0, -R_0)$. This changing of directions occurs every time a sweeper bumps into another. Each sweeper is responsible for an angular sector of the evader region that is proportional to the number of participating agents in the search. The search process can be viewed as a two-dimensional search in which the actual agents travel on a plane or as a three-dimensional search where the sweepers are dronelike agents that fly over the evader area. In case the search is three-dimensional, where sweepers fly at different heights above the evader region, every time a sweeper is directly above another, they exchange the angular section they are responsible to sweep between them, and continue the search. The analysis of the two cases is exactly the same.

Sweepers that employ a pincer movement solve the problem of evader region's spread from the "most dangerous points," points located at the tips of their sensors closest to the evader region's center. These points have the maximum time to spread during sweeper movement and, therefore, if evaders that try to escape from these points are detected, evaders trying to escape from other points are detected as well. When a sweeper returns to a location, the evader region has a smaller or equal radius than it had two cycles previously. If all sweepers were to rotate in the same direction after being deployed equally around the circle, the evader region's points that need to be considered for limiting the region's spread are points that are adjacent to the center of the sensor (for a circular sweep) and points that are adjacent to the sensors' tips that are furthest from the center of the evader region (for a spiral sweep). This consideration would lead to higher critical velocities for sweepers that employ same direction sweeps. Higher critical velocities also imply that, for a given sweeper velocity above the critical velocity, that is sufficient for both same direction and pincer-based sweep processes, sweep time is reduced when sweepers perform pincer movement sweep. In [28], the analysis of a single-agent circular sweep process indicates that the critical velocity for agents employing same direction sweeps is indeed higher compared to the pincer-based critical velocities developed in this article. In [29], this analysis is extended to study multiagent same-direction circular and spiral sweeping swarms and to compare the results to pincer-based search strategies. As discussed in Section VII of this article, the pincer-based strategies offer improved performance over their same-direction counterparts.

We analyze the proposed sweep processes' performance in terms of the total time to complete the search, defined as the time at which all potential evaders that resided in the initial evader region were detected. Expressions for the complete cleaning times of the evader region as a function of the search parameters, R_0 , r , V_T , and the number of agents n , in the swarm are derived, evaluated, and discussed for each developed sweep process. At first, we provide a global balance of covered areas argument derived from a maximal swept area versus a minimal danger zone expansion area for a given time interval. This argument results in an equation that yields a lower bound on a searcher velocity that is independent of the search process. Second, we examine the performance of a multiagent swarm that performs a proposed circular sweep process. A critical velocity that depends on this circular search process ensuring satisfaction of the confinement task is derived and compared to the lower bound on the critical velocity. We then show that the resulting circular critical velocity equals twice the lower bound and, hence, is not optimal. The purpose of designing a circular search process is to perform the task with simple agents, however, clearly it is not optimal. Therefore, the search pattern is improved, and a novel multiagent swarm spiral sweep process that uses spiral scans, drawing inspiration from a previous work of McGee and Hedrick [17], is proposed. The proposed pattern tracks the "wavefront" of the expanding evader region and strives to have optimal sensor footprint over the evader region. Based on this proposed search pattern, we obtain a new critical velocity that ensures the satisfaction of the confinement task for multiagent swarms. We then show that the spiral critical velocity approaches the theoretical optimal critical velocity that is independent of the search process. Finally, we compare the different search methods, circular and spiral, in terms of completion times of the sweep processes. When comparing the different search processes, we compare both the total cleaning times as well as the minimal searcher velocity required for a successful search.

Illustrative simulations that demonstrate the evolution of the search processes were generated using NetLogo software [30] and are presented in Figs. 2 and 3. Green areas are locations that are free from evaders and red areas indicate locations where potential evaders may still be located. Fig. 2 shows the cleaning progress of the evader region when six agents employ the circular pincer sweep process. Fig. 3 shows the cleaning progress of the evader region when four agents employ the spiral pincer sweep process.

In the considered problems, the exact locations of evaders and even their numbers are *a priori* unknown. The only information the sweepers have about the evaders' locations is that the evaders are located somewhere inside a given circular region at the beginning of the search process, and that the evaders may try to move and slip undetected out of this region as the search progresses to avoid interception. Since the sweepers do not have any additional knowledge about the evaders whereabouts, or even if all evaders were found at some intermediate point of time during the search, the search is continued until the whole region is searched, thus reducing the uncertainty region where potential evaders might be located to have an area of 0. Since finite environments of all shapes can be covered by a circle with a certain radius, a

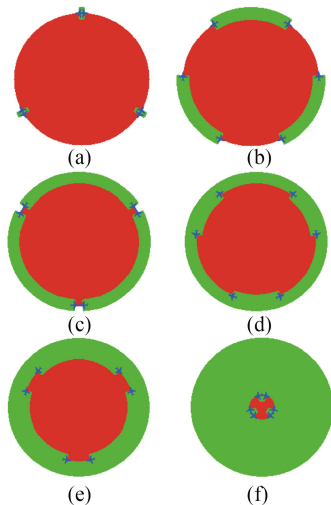


Fig. 2. Swept areas and evader region status for different times in a scenario where six agents employ the circular pincer sweep process. (a) Beginning of first cycle. (b) Midway of the first cycle. (c) Toward the completion of the first cycle. (d) Beginning of the second cycle. (e) Toward the end of the third cycle. (f) Beginning of the one before last cycle. Green areas are locations that are free from evaders and red areas indicate locations where potential evaders may still be located.

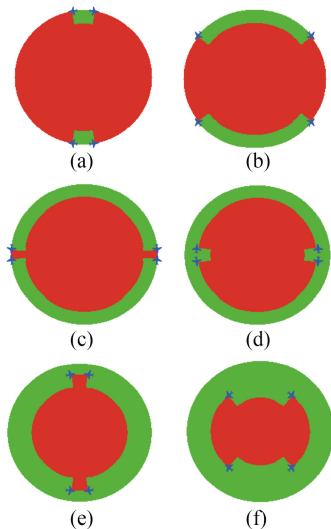


Fig. 3. Swept areas and evader region status for different times in a scenario where four agents employ the spiral pincer sweep process. (a) Beginning of first cycle. (b) Toward the midway of the first cycle. (c) Toward the completion of the first cycle. (d) Beginning of the second cycle. (e) Toward the end of the second cycle. (f) Toward the midway of the third cycle. Green areas are locations that are free from evaders and red areas indicate locations where potential evaders may still be located.

circle is chosen as a shape that generalizes such environments. If additional information, such as the number of evaders and their exact locations, is provided to the sweeper swarm in advance, the search time may be reduced by organizing the sweeper swarm differently, by changing the search pattern and by terminating the search at the point when an *a priori* known number of evaders are detected. Therefore, the resulting search times for a circular environment can be seen as an upper bound on the search time, resulting from the lack of specific information about evaders' locations. If more information is provided, the search time may

be reduced through the utilization of this knowledge to update and change the search process accordingly.

As opposed to our work, McGee and Hedrick [17] use a disk-shaped sensor with a radius of r , and does not calculate the time it takes to find all evaders. Furthermore, in our proposed sweep processes, the initial positions of the sweeper agents are different from the initial placements of agents in [17]. In [18], the searcher also uses a circular sensor of radius r that detects evaders if and only if they are at a distance of at most r from the searcher differing from the linear sensors used in our work. In [5] also, the searching agents move at a fixed velocity and evaders move at a velocity with a known limit, however the searchers are equipped with disk-shaped sensors and not linear detectors. With the same reasoning we use, Tang and Ozguner [5] guarantee that if evaders do not escape during the first traversal of the region, no escape occurs in subsequent traversals around a smaller region. As opposed to our proposed search patterns, where pairs of sweepers improve the trapping capabilities of the sweeper by utilizing pincer search trajectories, in [5], all searching agents move one after the other in the same direction. Furthermore, using the sweepers pincer motion enables us to avoid the complex end game we propose in [28] and allows the sweepers to clean the entire evader region using only circular and spiral sweeps.

Although the works in [22]–[24] are related to our research and use pincer movements between pairs of defenders as well, they have a different objective of protecting an initial region from entrance of invaders, unlike our goal that is to detect all evaders that may spread from the interior of the region. Furthermore, these works do not focus on guaranteeing interception of all evaders or intruders that try to enter the protected region but rather in devising policies for intercepting as many intruders as possible with agents that “crash” upon detection. These processes rely on an assumption that the number of intruders is finite and that each defender has to intercept only a single intruder. In the scenario, we investigate that the requirement is that all evaders are detected regardless of their number. Importantly, as opposed to our assumptions, where only the sweepers' sensors provide information on evaders' whereabouts, in the perimeter defense problems described in [22]–[24], the locations of intruders are known to the defenders, either throughout the whole scenario or from the time they are detected by a set of different patrolling agents. As opposed to the perimeter defense works mentioned earlier, at the beginning of every pincer maneuver the sweepers perform in our work, the placements of the sweeper pairs are back to back, preventing escape of evaders from the gap between the sweepers' sensors. Furthermore, the assignment of sweeping pairs in our work is not fixed and sweepers change their sweep partners during the search process in order to improve the sweeper team's performance.

III. UNIVERSAL BOUND ON CLEANING RATE

In this section, we present an optimal bound on the cleaning rate of a searcher with a linear-shaped sensor. This bound is independent of the particular search pattern employed. For each of the proposed search methods, we then compare the resulting

cleaning rate to the optimal derived bound in order to compare between different search methods. We denote the searcher's velocity as V_s , the sensor length as $2r$, the evader region's initial radius as R_0 , and the maximal velocity of an evading agent as V_T . The maximal cleaning rate occurs when the footprint of the sensor over the evader region is maximal. For a line-shaped sensor of length $2r$, this happens when the entire length of the sensor fully overlaps the evader region and it moves perpendicular to its orientation. The rate of sweeping when this happens has to be higher than the minimal expansion rate of the evader region (given its total area), otherwise no sweeping process can ensure detection of all evaders. We analyze the search process when the sweeper swarm is comprised of n identical agents. The smallest searcher velocity satisfying this requirement is defined as the critical velocity and denoted by V_{LB} , we have the following.

Theorem 1: No sweeping process is able to successfully complete the confinement task if its velocity V_s is less than

$$V_{LB} = \frac{\pi R_0 V_T}{nr}. \quad (1)$$

Proof: Denote by ΔT the interval of search. The maximal area that can be scanned when the searcher moves with a velocity V_s is given by

$$A_{\text{Max Clean}} = 2rV_s\Delta T. \quad (2)$$

Therefore, if each agent in the swarm of n cleans the maximal possible area, the maximal area that the swarm cleans is given by

$$A_{\text{Max Clean}}(n) = 2nrV_s\Delta T \quad (3)$$

i.e., the best cleaning rate is $2rnV_s$. The least spread of the evader region that expands due to evaders' possible motion with velocity V_T occurs when the region has the shape of a circle. This is due to the isoperimetric inequality: for a given area, the minimal boundary length that encloses it happens when the shape of the region is circular. Therefore, for an initial circular region with radius R_0 , the evader region minimal expansion is to a circle with a larger radius. For a spread of ΔT , the radius of the evader region grows to $R_0 + \Delta TV_T$ and the area of the evader region grows from πR_0^2 to $\pi(R_0 + \Delta TV_T)^2$. Therefore, the growth of the evader region area in time ΔT is $A_{\text{Least Spread}} = \pi(R_0 + \Delta TV_T)^2 - \pi R_0^2 = 2\pi R_0 \Delta TV_T + (\Delta TV_T)^2$. The spread rate is given by the division of the last expression by ΔT . Letting $\Delta T \rightarrow 0$, the expression resolves to $2\pi R_0 V_T$, the least possible spread rate. In order to guarantee the possibility of sweeping, we must set the best cleaning rate to be larger than the worst spread of area that is $2rnV_s \geq 2\pi R_0 V_T$. This yields the minimal velocity of a sweeper regardless of the search process it employs. Hence

$$V_s \geq \frac{\pi R_0 V_T}{nr} = V_{LB}. \quad (4)$$

Hopefully, after the first sweep, the evader region is within a circle with a smaller radius than the initial evader region's radius. Since the sweepers travel along the perimeter of the evader region and this perimeter decreases after the first sweep,

ensuring a sufficient sweeper velocity that guarantees that no evader escapes during the initial sweep guarantees also that the sweeper velocity is sufficient to prevent escape in subsequent sweeps as well. The formulation of the problem in terms of the smallest possible searcher velocity that guarantees a no escape search is equivalent to determining the maximal circular region that can be searched given a searcher's velocity of V_s , sensor length of $2r$, n sweepers, and a maximal velocity of an evading agent that is equal to V_T . Similar results for this bound for sweepers with circular sensors appear in [17].

IV. MULTIPLE AGENTS WITH LINEAR SENSORS: THE CIRCULAR PINCER SWEEP PROCESS

We analyze the case that the multiagent swarm consists of n agents, where n is an even number, and each sweeper has a sensor length of $2r$. At the beginning of the search process, the footprint of each sweeper's sensor that is over the evader region is equal to r . When employing this type of search pattern, the symmetry between the two agent trajectories prevents the escape from point $P = (0, R_0)$ that is the most dangerous point an evader can escape from as proved in [28]. Therefore, each sweeper's critical velocity is based only upon the time it takes it to traverse the angular section it responsible for, namely $\frac{2\pi}{n}$. For example, if the sweeper swarm consists of only two sweepers, each sweeper is required to scan an angle of π . If the sweepers' velocities are above the critical velocity of the scenario, the agents can advance inwards toward the center of the evader region after completing a cycle. The notion of a cycle or an iteration corresponds to an agent's traversal of the angular section it is required to scan, i.e., an angle of $\frac{2\pi}{n}$. Therefore, its definition varies with the number of sweepers. Once the agents finish scanning the angular section they are responsible for, and if their velocities allow it, they advance inwards together. In case the search is performed in the 2-D plane, the sweepers change their scanning direction only after the completion of an inward advancement maneuver. In case the search is three-dimensional, i.e., the sweepers fly over the evader region, the sweepers first advance inwards together, and only then exchange between them the angular section they are responsible to sweep. Afterward, the sweepers continue to scan a section with a smaller radius. Contrary to the works in [22]–[24] that devise methods for intercepting as many intruders as possible with agents that can intercept only a single invader, the circular pincer sweep process guarantees evaders are detected regardless of their number and the number of pursuing sweepers. Since each sweeper has a sensor length of r outside the evader region, in order to guarantee that no evader escapes the sweepers, we must demand that the spread of the evader region, from any potential location where an evader might be located, is confined to a radius of no more than r from its origin point at the beginning of the cycle. During an angular traversal of $\frac{2\pi}{n}$ around the evader region radius of R_0 , this yields that the following inequality must be satisfied:

$$\frac{2\pi R_0}{nV_s} \leq \frac{r}{V_T}. \quad (5)$$

Rearranging terms yields that the sweepers velocities must satisfy that

$$V_s \geq \frac{2\pi R_0 V_T}{nr}. \quad (6)$$

The critical velocity for the circular sweep process is therefore given when we have equality in (6)

$$V_c = \frac{2\pi R_0 V_T}{nr}. \quad (7)$$

Therefore, we obtain that the circular critical velocity equals twice the optimal minimal critical velocity

$$V_c = 2V_{LB}. \quad (8)$$

Theorem 2: For an n -agent swarm for which n is even that performs the circular sweep process, where the sweeper distribution is as described, the number of iterations it takes the swarm to reduce the evader region to be bounded by a circle with a radius that is less than or equal to r is given by

$$N_n = \left\lceil \frac{\ln \left(\frac{2\pi r V_T - nr V_s}{2\pi R_0 V_T - nr V_s} \right)}{\ln \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)} \right\rceil. \quad (9)$$

After N_n sweeps, the sweeper swarm performs an additional circular sweep and cleans the entire evader region.

We denote by T_{in} the sum of all inward advancement times and by $T_{circular}$ the sum of all the circular traversal times. Therefore, the time it takes the swarm to clean the entire evader region is given by

$$T(n) = T_{in}(n) + T_{circular}(n) \quad (10)$$

where $T_{in}(n)$ is given by

$$T_{in}(n) = \frac{R_0}{V_s} + \left(\frac{2\pi R_0 V_T - nr V_s}{n V_s (V_s + V_T)} \right) \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n - 1} \quad (11)$$

and $T_{circular}(n)$ is given by

$$T_{circular}(n) = -\frac{R_0(V_s + V_T)}{V_T V_s} + \frac{nr(V_s + V_T) + 2\pi r V_T}{2\pi V_T^2} + \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n} \left(\frac{(V_s + V_T)(2\pi R_0 V_T - nr V_s)}{2\pi V_s V_T^2} \right) + \frac{r(N_n - 1)}{V_T} + \frac{2\pi r}{n V_s}. \quad (12)$$

Proof: Let us denote by $\Delta V > 0$ the addition to the sweeper's velocity above the critical velocity. The sweeper's velocity is therefore given by $V_s = V_c + \Delta V$. The time it takes each sweeper to circularly sweep the region it is responsible to sweep is given by

$$T_{circular_i} = \frac{2\pi R_i}{n(V_c + \Delta V)}. \quad (13)$$

Substituting $V_s = V_c + \Delta V$ in (13) yields

$$T_{circular_i} = \frac{2\pi R_i}{n V_s}. \quad (14)$$

As a function of the number of sweepers and the iteration number, the distance a sweeper can advance inwards after completing an iteration is given by

$$\delta_i(\Delta V) = r - V_T T_{circular_i}, \quad 0 \leq \delta_i(\Delta V) \leq r \quad (15)$$

where in the term $\delta_i(\Delta V)$, ΔV denotes the increase in the agent's velocity relative to the critical velocity. The number of sweep iterations the sweeper performed around the evader region is denoted by i , where i starts from sweep number 0. After the agents complete a cycle, they move inwards toward the center of the evader region in a way that the inner tips of their sensors point toward the center of evader region. When the sweepers progress inwards, they progress with a velocity of V_s , until they reach the position where they start their next sweep at the moment they meet the evader region's expanding wavefront. During the inwards advancements, no cleaning is performed while the evader region continues to spread.

The time it takes the sweepers to move inwards until half of their sensors are over the evader region depends on the relative velocity between the agents inwards entry and the evader region outwards expansion and is given by (17). As the sweepers progress toward the center of the evader region, the evader region continues to expand. Therefore, sweepers can only advance by a smaller distance than $\delta_i(\Delta V)$, denoted by $\delta_{i,eff}(\Delta V)$, which depends on the ratio between the velocity in which the sweepers progress toward the center of the region and the sum of velocities of sweeper and evader region spread. $\delta_{i,eff}(\Delta V)$ is the actual distance sweepers move at each iteration in order to meet the wavefront of the evader region when half of their sensors are covered by the evader region. Therefore, the distance an agent can advance inwards after completing an iteration is given by

$$\delta_{i,eff}(\Delta V) = \delta_i(\Delta V) \left(\frac{V_s}{V_s + V_T} \right). \quad (16)$$

The inward advancement time depends on the iteration number. It is denoted by T_{in_i} and is given by

$$T_{in_i} = \frac{\delta_{i,eff}(\Delta V)}{V_s} = \frac{r n V_s - 2\pi R_i V_T}{n V_s (V_s + V_T)}. \quad (17)$$

The index i in T_{in_i} denotes the iteration number in which the advancement is done. After all agents complete their sweep, the evader region is bounded by a circle with a smaller radius compared to the previous sweep. Thus, the new radius of the circle that bounds the evader region is given by

$$R_{i+1} = R_i - \delta_{i,eff}(\Delta V) = R_i - \delta_i(\Delta V) \left(\frac{V_s}{V_s + V_T} \right). \quad (18)$$

Plugging the value of $\delta_i(\Delta V)$ from (15) into (18) yields

$$R_{i+1} = R_i - \delta_{i,eff}(\Delta V) = R_i - \frac{r V_s}{V_s + V_T} + \frac{2\pi R_i V_T}{n(V_s + V_T)}. \quad (19)$$

Rearranging terms yields

$$R_{i+1} = R_i \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right) - \frac{r V_s}{V_s + V_T}. \quad (20)$$

The exact value of R_N can only be calculated after the number of sweeps around the region N_n is calculated. Therefore, we use $\hat{R}_N = r$ as an estimate of the true R_N in order to calculate N_n . For any even number of sweepers n , the search continues in this way until the evader region is confined to a radius of $\hat{R}_N = r$.

Denoting the coefficients c_1 and c_2 by

$$c_1 = -\frac{rV_s}{V_s + V_T}, c_2 = 1 + \frac{2\pi V_T}{n(V_s + V_T)}. \quad (21)$$

Equation (20) takes the form of

$$R_{i+1} = c_2 R_i + c_1. \quad (22)$$

The number of iterations it takes the sweeper swarm to reduce the evader region to be bounded by a circle with a radius of $\hat{R}_N = r$ that corresponds to the last sweep before completely cleaning the evader region is calculated in Appendix A. It is given by

$$N_n = \left\lceil \frac{1}{\ln c_2} \ln \left(\frac{\hat{R}_N - \frac{c_1}{1-c_2}}{R_0 - \frac{c_1}{1-c_2}} \right) \right\rceil. \quad (23)$$

Substitution of coefficients in (23) yields that the number of iterations it takes the sweepers to reduce the evader region to be contained in a circle with the radius of the last scan $\hat{R}_N = r$ is given by

$$N_n = \left\lceil \frac{\ln \left(\frac{2\pi r V_T - nr V_s}{2\pi R_0 V_T - nr V_s} \right)}{\ln \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)} \right\rceil. \quad (24)$$

The ceiling operator is applied since the number of iterations has to be an integer number in order to allow the sweepers to complete scanning the sections they sweep and meet with the sweeper that scans the adjacent section. This practically implies that the sweepers continue the N_n th sweep, even if the evader region's radius slightly decreases below r during the sweep.

The total time it takes a multiagent swarm of n sweepers to clean the evader region is given by the total time of inward advancements combined with the total circular traversal times around the evader region in all cycles. We denote by $T_{\text{in}}(n)$ the sum of all the inward advancement times and by $T_{\text{circular}}(n)$ the sum of all the circular traversal times. Namely

$$T(n) = T_{\text{in}}(n) + T_{\text{circular}}(n). \quad (25)$$

We denote the total advancement time until the evader region is bounded by a circle with a radius that is less than or equal to r as $\tilde{T}_{\text{in}}(n)$. It is given by

$$\tilde{T}_{\text{in}}(n) = \sum_{i=0}^{N_n-2} T_{\text{in}_i}. \quad (26)$$

During the inward advancements, only the tip of the sensor, which has zero width, is inserted into the evader region. Therefore, no evaders are detected until the sweeper completes its inward advance and starts sweeping again. After the sweeper completes its advance into the evader region, its sensor footprint over the evader region is equal to r . The total search time until the evader region is bounded by a circle with a radius that is less than or equal to r is given by the sum of the total circular sweep times and the times of the inward advances. Namely

$$\tilde{T}(n) = \tilde{T}_{\text{in}}(n) + \tilde{T}_{\text{circular}}(n). \quad (27)$$

Using the developed term for T_{in_i} , the total inward advancement times until the evader region is bounded by a circle with a radius

that is less than or equal to r are computed by

$$\tilde{T}_{\text{in}}(n) = \sum_{i=0}^{N_n-2} T_{\text{in}_i} = \frac{(N_n - 1)r}{V_s + V_T} - \frac{2\pi V_T}{nV_s(V_s + V_T)} \sum_{i=0}^{N_n-2} R_i. \quad (28)$$

We note that the first inward advancement occurs when the evader region is bounded by a circle of radius R_0 and the last inward advancement occurs at iteration number $N_n - 2$, which describes the inward advancement in which the evader region transitions from being bounded by a circle of radius R_{N_n-2} to being bounded by a circle of radius R_{N_n-1} . Afterward, the sweeper swarm completes another circular sweep where after its completion, the evader region is bounded by a circle of radius R_N . The calculation is done in this way since at the last sweep, the sweeping agents advance a distance that is equal to or smaller than the allowable distance they can advance toward the center of the evader region in order to ensure that their paths do not cross each other. Therefore, in order to prevent collisions between the sweeping agents at the last iteration before they completely clean the evader region, we ensure that the lower tips of the sweepers' sensors do not cross the center of the evader region. The full derivation of $\tilde{T}_{\text{in}}(n)$ can be found in Appendix F. This derivation yields that

$$\begin{aligned} \tilde{T}_{\text{in}}(n) &= \sum_{i=0}^{N_n-2} T_{\text{in}_i} \\ &= \frac{R_0}{V_s} - \frac{nr}{2\pi V_T} - \left(1 + \frac{2\pi V_T}{n(V_s + V_T)}\right)^{N_n-1} \left(\frac{2\pi R_0 V_T - nr V_s}{2\pi V_T V_s}\right). \end{aligned} \quad (29)$$

In order to calculate $T_{\text{in}}(n)$, we must add the last inward advancement. This time is given by $T_{\text{in}_{\text{last}}}(n) = \frac{R_{N_n}}{V_s}$. Therefore

$$T_{\text{in}_{\text{last}}}(n) = \frac{nr}{2\pi V_T} + \left(1 + \frac{2\pi V_T}{n(V_s + V_T)}\right)^{N_n} \left(\frac{2\pi R_0 V_T - nr V_s}{2\pi V_T V_s}\right). \quad (30)$$

$T_{\text{in}}(n)$ is given as $T_{\text{in}}(n) = \tilde{T}_{\text{in}}(n) + T_{\text{in}_{\text{last}}}(n)$ and, therefore, yields

$$T_{\text{in}}(n) = \frac{R_0}{V_s} + \left(\frac{2\pi R_0 V_T - nr V_s}{nV_s(V_s + V_T)}\right) \left(1 + \frac{2\pi V_T}{n(V_s + V_T)}\right)^{N_n-1}. \quad (31)$$

We now proceed to the calculation of the circular sweep times. The initial circular sweep time is given by $T_0 = \frac{2\pi R_0}{nV_s}$. The relation between the time to circularly sweep a circle of radius R_i by an angle of $\frac{2\pi}{n}$ at a velocity of V_s is given by

$$T_i = \frac{2\pi R_i}{nV_s}. \quad (32)$$

We denote the coefficient c_3 by

$$c_3 = -\frac{2\pi r}{n(V_s + V_T)}. \quad (33)$$

Multiplying (22) by $\frac{2\pi}{nV_s}$ yields a recursive difference equation for the sweep times. Hence, the sweep times are expressed as

$$T_{i+1} = c_2 T_i + c_3. \quad (34)$$

Let $\tilde{T}_{\text{circular}}(n)$ denote the sum of circular sweep times until the evader region is bounded by a circle that is less than or equal to

r . $\tilde{T}_{\text{circular}}(n)$ is developed in Appendix C and is given by

$$\tilde{T}_{\text{circular}}(n) = \frac{T_0 - c_2 T_{N_n-1} + (N_n - 1) c_3}{1 - c_2}. \quad (35)$$

The last circular sweep time before the evader region is bounded by a circle with a radius smaller or equal to r is computed in Appendix D and is given by

$$T_{N_n-1} = \frac{c_3}{1 - c_2} + c_2^{N_n-1} \left(T_0 - \frac{c_3}{1 - c_2} \right). \quad (36)$$

Plugging the respective coefficients into (36) yields

$$T_{N_n-1} = \frac{r}{V_T} + \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n-1} \left(\frac{2\pi R_0 V_T - r n V_s}{n V_s V_T} \right). \quad (37)$$

Substituting the coefficients in (35) with the respective developed terms yields

$$\begin{aligned} \tilde{T}_{\text{circular}}(n) = & -\frac{R_0(V_s + V_T)}{V_T V_s} + \frac{nr(V_s + V_T) + 2\pi r V_T}{2\pi V_T^2} \\ & + \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n} \left(\frac{(V_s + V_T)(2\pi R_0 V_T - r n V_s)}{2\pi V_s V_T^2} \right) + \frac{r(N_n - 1)}{V_T}. \end{aligned} \quad (38)$$

After the completion of sweep N_n , the evader region is bounded by a circle with a radius that is less than or equal to r . In order to prevent the paths of the sweepers from coinciding at the last sweep, the sweepers advance toward the center of the evader region until the lower tips of their sensors are at the center of the evader region. Following this advancement, they perform the last circular sweep. The time to perform this sweep is denoted by $T_{\text{last}}(n)$. $T_{\text{last}}(n)$ is the time it takes the sweepers to complete the last circular sweep of radius r while traversing an angle of $\frac{2\pi}{n}$ around the center of the evader region. $T_{\text{last}}(n)$ is given by $T_{\text{last}}(n) = \frac{2\pi r}{n V_s}$. Therefore, the total time of circular sweeps until complete cleaning of the evader region is given by

$$T_{\text{circular}}(n) = \tilde{T}_{\text{circular}}(n) + T_{\text{last}}(n). \quad (39)$$

Hence

$$\begin{aligned} T_{\text{circular}}(n) = & -\frac{R_0(V_s + V_T)}{V_T V_s} + \frac{nr(V_s + V_T) + 2\pi r V_T}{2\pi V_T^2} \\ & + \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n} \left(\frac{(V_s + V_T)(2\pi R_0 V_T - r n V_s)}{2\pi V_s V_T^2} \right) \\ & + \frac{r(N_n - 1)}{V_T} + \frac{2\pi r}{n V_s}. \end{aligned} \quad (40)$$

Fig. 4 depicts the cleaning progress of the evader region when 4 agents employ the circular pincer sweep process. Fig. 5 presents the complete cleaning times for different numbers of sweepers. The total time comprises the sum of circular sweep times and the inward advancement times. In each of the plotted curves, the velocity of the sweepers is equal, regardless of the number of sweepers that perform the search, in order to make a fair comparison between cleaning times of swarms with different number of sweepers. ΔV is taken with respect to the critical velocity of two sweepers, since, as the number of sweepers increases, the critical velocity that enables them to complete the search decreases. Therefore, swarms with increasing number of sweepers are able to complete the search when moving at a velocity that is above the critical velocity of two sweepers while the converse does not necessarily hold. Fig. 6 presents

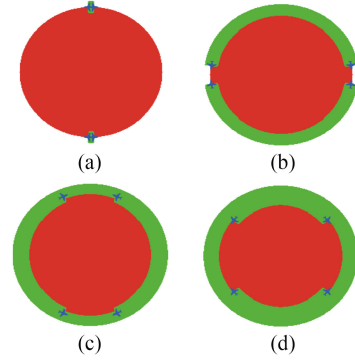


Fig. 4. Swept areas and evader region status for different times in a scenario where four agents employ the circular pincer sweep process. (a) Beginning of first cycle. (b) Toward the end of the first cycle. (c) Toward the end of the second cycle. (d) Beginning of the third cycle. Green areas are locations that are free from evaders and red areas indicate locations where potential evaders may still be located.

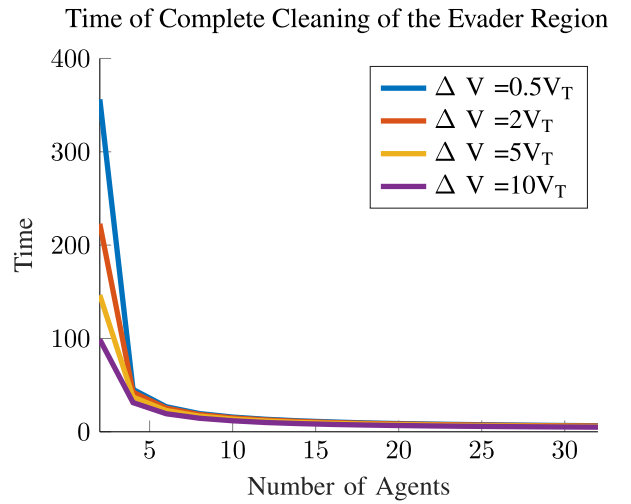


Fig. 5. Time of complete cleaning of the evader region. We simulated the circular pincer sweep processes for an even number of agents, ranging from 2 to 32 agents. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

Sum of Times of Circular Sweeps

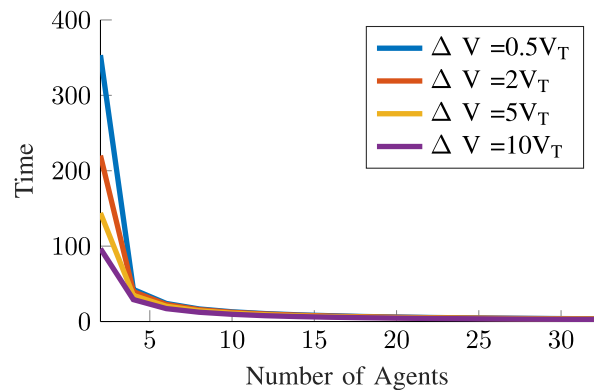


Fig. 6. Sum of circular sweep times of the search until complete cleaning of the evader region. We simulated circular sweep processes with an even number of agents, ranging from 2 to 32 agents. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

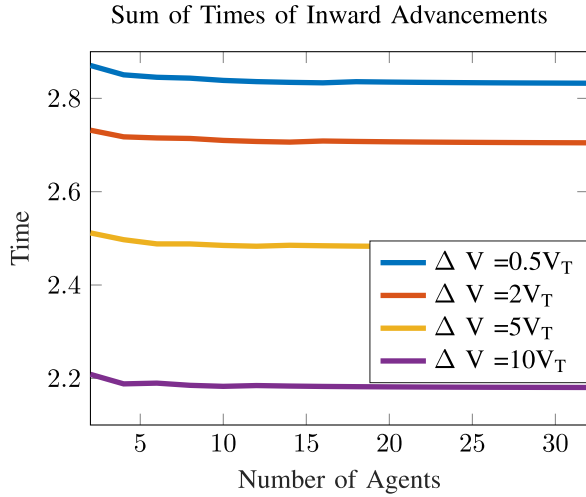


Fig. 7. Sum of inward advancement times until complete cleaning of the evader region. We simulated circular sweep processes with an even number of agents, ranging from 2 to 32 agents. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

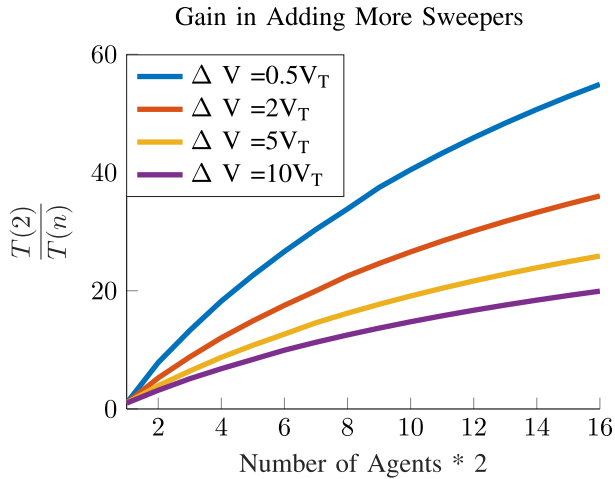


Fig. 8. Cleaning time gain obtained by adding sweepers. We simulated circular sweep processes with an even number of agents, denoted by n , ranging from 2 to 32 agents. In each curve, every point is obtained by the ratio between the sweep times of a two agent swarm and an n agent swarm. We show results obtained for different values of velocities above the circular critical velocity, i.e., different choices for ΔV . The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

only the circular cleaning times and Fig. 7 presents only the inward advancement times until the complete cleaning of the evader region. Fig. 8 shows that the gain in search time reduction is more pronounced when velocities are closer to the critical velocity.

V. MULTIPLE AGENTS WITH LINEAR SENSORS: THE SPIRAL PINCER SWEEP PROCESS

Since at the start of every circular sweep process, half of the sweepers' sensors are outside of the evader region, we would like the sweepers to employ a more efficient motion throughout the cleaning process. This means that throughout the motion of the searcher, the footprint of its sensor that is above the evader region should maximally overlap the evader region. This can be

achieved with a spiral scan, where the agent's sensor tracks the expanding evader region wavefront while preserving its shape to be as close as possible to a circle.

We analyze the case that the multiagent swarm consists of n agents, where n is an even number, and each sweeper has a line sensor length of $2r$, choosing all sweepers to have sensors of equal lengths implies that when the sweepers' sensors reach the same point and are tangent to each other, escape will not be possible from the gap between the sensors. This is a necessary requirement since the process that is described below relies on the symmetry between the agents' sensors that are over the evader region.

At the beginning of the search process, the footprint of each sweeper's sensor that is over the evader region is equal to $2r$. When employing this type of search pattern, the symmetry between the two agent trajectories prevents the escape from point $P = (0, R_0)$ that is the most dangerous point an evader can escape from. Therefore, each sweeper's critical velocity can be based only upon the time it takes it to traverse the angular section it responsible for, namely $\frac{2\pi}{n}$. Similar to the circular sweep process, when sweepers move at a velocity greater than their critical velocity and if the search is carried out on the 2-D plane, sweepers change their direction of scanning after performing an inward advancement. If the search is three-dimensional, the sweepers advance inwards together, exchanging afterward between them the angular section they are responsible to sweep. After this motion, the sweepers start scanning a section with a smaller radius.

Each searcher begins its spiral traversal with the tip of its sensor tangent to the edge of the evader region. In order to keep its sensor tangent to the evader region throughout the sweep, the searcher must travel at an angle ϕ to the normal of the evader region. ϕ depends on the ratio between the sweeper and evader velocities. The incentive of a sweeper to travel in a constant angle ϕ to the normal of the evader region is to preserve the evader region's circular shape and keep the entire length of its sensor inside the evader region at all times. Fig. 1(b) shows an initial placement of two agents that employ the pincer movement spiral sweep process. ϕ is calculated by

$$\sin \phi = \frac{V_T}{V_s}. \quad (41)$$

Thus, we have

$$\phi = \arcsin \left(\frac{V_T}{V_s} \right). \quad (42)$$

The isoperimetric inequality states that for a given area, the shape of the curve that bounds the area which has the smallest perimeter is circular. Since the agent travels along the perimeter of the evader region while keeping the evader region's shape close to circular, the isoperimetric inequality implies that the time it takes to complete a sweep around the region is minimal. The agent's angular velocity or rate of change of its angle with respect to the center of the evader region θ_s can be described as a function of ϕ as

$$\frac{d\theta_s}{dt} = \frac{V_s \cos \phi}{R_s(t)} = \frac{\sqrt{V_s^2 - V_T^2}}{R_s(t)}. \quad (43)$$

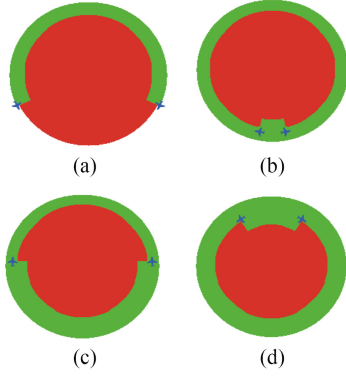


Fig. 9. Swept areas and evader region status for different times in a scenario where two agents employ the spiral pincer sweep process. (a) Toward the completion of the first cycle. (b) Beginning of the second cycle. (c) Midway of the second cycle. (d) Beginning of the third cycle. Green areas are locations that are free from evaders and red areas indicate locations where potential evaders may still be located.

The instantaneous growth rate of the searcher's radius is given by

$$\frac{dR_s(t)}{dt} = V_s \sin \phi = V_T. \quad (44)$$

Integrating (43) between the initial and final sweep times of the angular section yields

$$\int_0^{t_\theta} \dot{\theta}(\zeta) d\zeta = \int_0^{t_\theta} \frac{\sqrt{V_s^2 - V_T^2}}{V_T \zeta + R_0 - r} d\zeta. \quad (45)$$

The result of the integral in (45) yields

$$\theta(t_\theta) = \frac{\sqrt{V_s^2 - V_T^2}}{V_T} \ln \left(\frac{V_T t_\theta + R_0 - r}{R_0 - r} \right). \quad (46)$$

Applying the exponent function to (46) results in

$$(R_0 - r) \exp \left(\frac{V_T \theta(t_\theta)}{\sqrt{V_s^2 - V_T^2}} \right) = V_T t_\theta + R_0 - r = R_s(t_\theta). \quad (47)$$

Each sweeper begins its spiral traversal with the tip of its sensor tangent to the edge of the evader region. Fig. 1(b) shows an example for the initial placement of two agents where the tips of the sweepers' sensors are located at point $P = (0, R_0)$. Fig. 9 shows the cleaning progress of the evader region when two agents employ the spiral sweep process. The time it takes the sweeper to complete a spiral traversal around the angular region of the evader region it is responsible to scan corresponds to changing its angle θ by $\frac{2\pi}{n}$. During this time, the expansion of the evader region has to be by no more than $2r$ from its initial radius, in order for the sweeper to prevent the escape of potential evaders. This assertion holds under the assumption that after each cycle when the sweeper advances inwards toward the center of the evader region, it completes this motion in zero time. Otherwise, the spread of evaders has to be less than $2r$ and considerations, such as the spread of evaders during the inwards motion, needs to be taken into account. This case is addressed after the analysis of the simplified case that is described here. In order for no evader to escape the sweepers, after a traversal of

$\frac{2\pi}{n}$, the following inequality must hold:

$$R_0 + r \geq R_s \left(t \frac{2\pi}{n} \right). \quad (48)$$

Define

$$\lambda = \exp \left(\frac{2\pi V_T}{n \sqrt{V_s^2 - V_T^2}} \right). \quad (49)$$

Substituting $R_s \left(t \frac{2\pi}{n} \right)$ with the expression of the trajectory of the center of the sweeper yields

$$R_0 + r \geq (R_0 - r) \lambda. \quad (50)$$

Therefore, in order to ensure no evader escapes undetected during the sweeping maneuver, the sweepers' velocities must satisfy

$$V_s \geq V_T \sqrt{\frac{\left(\frac{2\pi}{n} \right)^2}{\left(\ln \left(\frac{R_0 + r}{R_0 - r} \right) \right)^2} + 1}. \quad (51)$$

We now propose a modification to the construction of the critical velocity given in (51). This modification takes into account the consideration that when the sweepers travel toward the center of the evader region after completing the spiral sweep, they have to meet the evader wavefront travelling outwards the region with a speed of V_T at the previous radius R_0 . This more realistic update of the search process makes the spiral sweep process critical velocity agree with the optimal lower bound on the sweeper velocity that is independent of the sweep process and is slightly above it. The derived critical velocity ensures that no evaders escape while the sweepers advance toward the center of the evader region. We have that the expansion of the evader region during the first sweep denoted by T_c has to satisfy that $V_T T_c \leq \frac{2r V_s}{V_s + V_T}$. Substituting the expression for T_c yields

$$(R_0 - r) (\lambda - 1) = \frac{2r V_s}{V_s + V_T}. \quad (52)$$

Corollary 1: For an n -agent spiral sweep process, where n is even, the critical velocity V_s , allowing the satisfaction of the confinement task, is obtained as the solution of

$$V_T T_c = \frac{2r V_s}{V_s + V_T} \quad (53)$$

where T_c is given by

$$T_c = \frac{(R_0 - r) (\lambda - 1)}{V_T}. \quad (54)$$

The modified critical velocity of Corollary 1 is computed numerically using the Newton–Raphson method with the velocity derived in (51) as an initial guess. All forthcoming derivations use the modified critical velocity of Corollary 1 that accounts for the spread of evaders during the sweepers' advancements toward the center of the evader region.

As shown in Fig. 10, the computed spiral critical velocity is close to the optimal lower bound, especially for a small number of sweepers. For example, for the two sweeper case, the ratio between the spiral critical velocity and the optimal velocity is 1.05. Since the number of evaders and their exact locations is

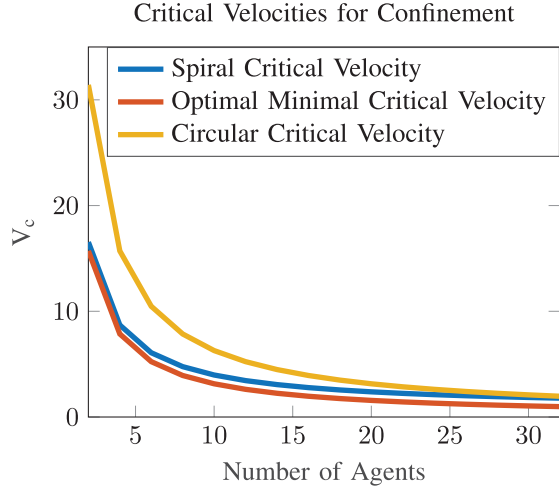


Fig. 10. Critical velocities as a function of the number of sweepers. The number of sweeper agents is even, ranges from 2 to 32 agents, which employ the multiagent spiral sweep process where the inward advancements toward the center of the evader are taken into account. We also plot the optimal critical velocities and the circular critical velocities for comparison. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

unknown, if sweepers allow an evader to escape from the region they sweep, with the intention of enabling themselves to move at a lower velocity than the critical velocity, there is no guarantee that all evaders in the region will be detected. If such a pair of sweepers allows an evader to cross the region they sweep undetected, even by coordinating with another sweeper that will chase this evader and detect it later, evaders that are inside the chasing sweeper's allocated region may take advantage of this movement and use this opportunity to escape, resulting in the failure of the mission.

Theorem 3: For an n -agent swarm for which n is even, which performs the spiral sweep process, where the sweepers distribution is as described, the number of iterations it takes the swarm to clean the entire evader region is given by

$$\tilde{N}_n = N_n + \eta + 1 = \left\lceil \frac{\ln \left(\frac{r(3-\lambda)}{R_0(1-\lambda) + r(1+\lambda)} \right)}{\ln \left(\frac{V_T + V_s \lambda}{V_s + V_T} \right)} \right\rceil + \eta + 1 \quad (55)$$

where $\eta = 0$ or $\eta = 1$. We denote by $T_{in}(n)$ the sum of all the inward advancement times and by $T_{spiral}(n)$ the sum of all the spiral traversal times. Therefore, the time it takes the swarm to clean the entire evader region is given by

$$T(n) = T_{in}(n) + T_{spiral}(n) \quad (56)$$

where $T_{in}(n)$ is given by

$$T_{in}(n) = \tilde{T}_{in}(n) + T_{inlast}(n) + \eta T_{inf}(n) \quad (57)$$

where $\tilde{T}_{in}(n)$ is given by

$$\tilde{T}_{in}(n) = \frac{2r}{V_s + V_T} + \frac{R_0 - r}{V_s} + \frac{2r(V_T + V_s \lambda)}{V_s(V_s + V_T)(1-\lambda)} - \frac{(V_T + V_s \lambda)^{N_n - 1}}{V_s(V_s + V_T)(1-\lambda)} (R_0(1-\lambda) + r(1+\lambda)). \quad (58)$$

$T_{inlast}(n)$ is given by $T_{inlast}(n) = \frac{R_N}{V_s}$ and $T_{inf}(n)$ is given by $T_{inf}(n) = \frac{T_i V_T}{V_s}$. Therefore

$$T_{in}(n) = \tilde{T}_{in}(n) + \frac{R_N}{V_s} + \frac{\eta r}{V_s} (\lambda - 1). \quad (59)$$

$T_{spiral}(n)$ is given by

$$T_{spiral}(n) = \tilde{T}_{spiral}(n) + T_{last}(n) + \eta T_l(n) \quad (60)$$

where $\tilde{T}_{spiral}(n)$ is given by

$$\tilde{T}_{spiral}(n) = \frac{(r-R_0)(V_s+V_T)}{V_T V_s} - \frac{2r(V_T+V_s \lambda)}{V_T V_s (1-\lambda)} + \frac{2r(N_n-1)}{V_T} - \left(\frac{V_T+V_s \lambda}{V_s+V_T} \right)^{N_n} \left(\frac{(V_s+V_T)(R_0(\lambda-1)-r(\lambda+1))}{V_T V_s (1-\lambda)} \right). \quad (61)$$

$T_{last}(n)$ is given by $T_{last}(n) = \frac{2\pi r}{n V_s}$ and $T_l(n)$ is given by $T_l(n) = \frac{r(\lambda-1)}{V_T}$. Hence, $T_{spiral}(n)$ is given by

$$T_{spiral}(n) = \tilde{T}_{spiral}(n) + \frac{2\pi r}{n V_s} + \eta \frac{r(\lambda-1)}{V_T}. \quad (62)$$

Proof: Let us denote by $\Delta V > 0$ the addition to the sweeper's velocity above the critical velocity. The sweeper's velocity is therefore given by $V_s = V_c + \Delta V$. The expression of the angle the sweeper travels with respect to the center of evader region is denoted as $\theta(t_\theta)$. At the beginning of the cycle, the center of the sweeper's sensor is located at a distance of $R_i - r$ from the center of the evader region. $\theta(t_\theta)$ is calculated in (46). Replacing R_0 with R_i yields

$$\theta(t_\theta) = \frac{\sqrt{V_s^2 - V_T^2}}{V_T} \ln \left(\frac{V_T t_\theta + R_i - r}{R_i - r} \right). \quad (63)$$

The time it takes the sweeper to travel an angle of $\theta(t_\theta) = \frac{2\pi}{n}$ is denoted as T_{spiral_i} and is obtained from (63). It is given by

$$T_{spiral_i} = \frac{(R_i - r)(\lambda - 1)}{V_T}. \quad (64)$$

Given that an agent moves in a velocity that is greater than the critical velocity for the corresponding scenario, we denote as in the circular sweep process, the distance an agent can advance toward the center of the evader region by $\delta_i(\Delta V)$. After the completion of the inward advancement movement, the evader region shrinks to an updated circular evader region with a radius of $R_{i+1} = R_i - \delta_i(\Delta V)$. After completing the proposed spiral sweep, the evader region is again circularly shaped, with a smaller radius. A proof for this property is provided in Appendix H. We have that

$$\delta_i(\Delta V) = 2r - V_T T_{spiral_i}, \quad 0 \leq \delta_i(\Delta V) \leq 2r. \quad (65)$$

As a function of the number of sweepers and iteration number, the distance sweepers can advance inwards after completing an iteration in case the evader wavefront did not continue to expand during the sweepers' inward motion is given by

$$\delta_i(\Delta V) = 2r - (R_i - r)(\lambda - 1) \quad (66)$$

where in the term $\delta_i(\Delta V)$, ΔV denotes the increase in the agent's velocity relative to the critical velocity. The number of sweep iterations the sweepers performed around the evader

region is denoted by i , where i starts from sweep number 0. As in the case of the circular sweep process, the time it takes the sweepers to move inwards until their entire sensors are over the evader region depends on the relative velocity between the sweepers inwards entry velocities and the evader region outwards expansion velocity. Therefore, the distance a sweeper advances inwards after completing an iteration is given by

$$\delta_{i\text{eff}}(\Delta V) = \delta_i(\Delta V) \left(\frac{V_s}{V_s + V_T} \right). \quad (67)$$

The new radius of the smaller circular evader region is therefore given by

$$R_{i+1} = R_i - \delta_i(\Delta V) \left(\frac{V_s}{V_s + V_T} \right). \quad (68)$$

We denote by $\tilde{R}_i = R_i - r$. Using \tilde{R}_i instead of R_i yields equations that have the same structure as the equations that were developed for the circular sweep process. This allows to solve the resulting spiral sweep process's equations with the same methodology along with the appropriate change of coefficients. Substituting the value for $\delta_i(\Delta V)$ into (68) results in

$$\tilde{R}_{i+1} = \tilde{R}_i - \left(2r - \tilde{R}_i(\lambda - 1) \right) \left(\frac{V_s}{V_s + V_T} \right). \quad (69)$$

Rearranging terms yields a difference equation with similar structure to the difference equation that was obtained for the circular sweep process

$$\tilde{R}_{i+1} = \tilde{R}_i \left(\frac{V_T + V_s\lambda}{V_s + V_T} \right) - \frac{2rV_s}{V_s + V_T}. \quad (70)$$

Denoting the coefficients in (70) as

$$c_1 = -\frac{2rV_s}{V_s + V_T}, c_2 = \frac{V_T + V_s\lambda}{V_s + V_T} \quad (71)$$

yields the following difference equation:

$$\tilde{R}_{i+1} = c_2\tilde{R}_i + c_1. \quad (72)$$

R_N is the actual radius of the circular evader region whose radius is smaller or equal to $2r$ and is calculated by similar steps as R_{N-2} is calculated in Appendix B. The precise calculation of R_N is important for the end game of the cleaning process, as is discussed later in this section. The exact value of R_N can only be calculated after the number of sweeps around the region N_n is calculated. Therefore, we use $\hat{R}_N = 2r$ as an estimate of the true R_N in order to calculate N_n .

Since the structure of the difference equation for the radius of the evader region is the same as those in the circular case described in the previous section, the number of iterations it takes the sweepers to reduce the evader region to a circle of radius $\hat{R}_N = 2r$ is given by

$$N_n = \left\lceil \frac{1}{\ln c_2} \ln \left(\frac{\hat{R}_N - \frac{c_1}{1-c_2}}{R_0 - \frac{c_1}{1-c_2}} \right) \right\rceil. \quad (73)$$

The ceiling operator is applied since the number of iterations has to be an integer number in order to allow the sweepers to complete scanning the sections they sweep. This practically implies that the sweepers continue sweep number N_n even if

the evader region's radius slightly decreases below $2r$ during the sweep. Therefore, in order to calculate N_n , we assume the last cycle takes place when the evader region is a circle of radius $\hat{R}_N = 2r$ or $\hat{R}_N = r$. Thus, substitution of coefficients in (73) yields that after N_n iterations, the evader region is circularly shaped with a radius that is less than or equal to $2r$. N_n is given by

$$N_n = \left\lceil \frac{\ln \left(\frac{r(3-\lambda)}{R_0(1-\lambda)+r(1+\lambda)} \right)}{\ln \left(\frac{V_T+V_s\lambda}{V_s+V_T} \right)} \right\rceil. \quad (74)$$

The inwards advancement time depends on the iteration number. It is denoted by T_{in_i} and its expression is given by

$$T_{\text{in}_i} = \frac{\delta_{i\text{eff}}(\Delta V)}{V_s} = \frac{2r - \tilde{R}_i(\lambda - 1)}{V_s + V_T}. \quad (75)$$

We denote the total advancement time until the evader region is reduced to a circle of radius that is less than or equal to $2r$ as $\tilde{T}_{\text{in}}(n)$. It is given by $\tilde{T}_{\text{in}}(n) = \sum_{i=0}^{N_n-2} T_{\text{in}_i}$.

Since during the inward advancements, only the tip of the sensor, which has zero width, is inserted into the evader region, it does not detect evaders until the sweeper completes its inward advance and starts sweeping again. After the sweepers complete their advance into the evader region, their sensor footprint over the evader region is equal to $2r$. The total search time until the evader region is reduced to a circle with a radius that is less than or equal to $2r$ is given by the sum of the total spiral sweep times combined with the times of the total inward advancements. Namely

$$\tilde{T}(n) = \tilde{T}_{\text{in}}(n) + \tilde{T}_{\text{spiral}}(n). \quad (76)$$

Using the developed term for T_{in_i} , the total inward advancement times until the evader region is reduced to a circle with a radius that is less than or equal to $2r$ are computed by

$$\tilde{T}_{\text{in}}(n) = \sum_{i=0}^{N_n-2} T_{\text{in}_i} = \sum_{i=0}^{N_n-2} \frac{2r - \tilde{R}_i(\lambda - 1)}{V_s + V_T}. \quad (77)$$

The full derivation of $\tilde{T}_{\text{in}}(n) = \sum_{i=0}^{N_n-2} T_{\text{in}_i}$ is given in Appendix G. We therefore have that

$$\tilde{T}_{\text{in}}(n) = \frac{2r}{V_s + V_T} + \frac{R_0 - r}{V_s} + \frac{2r(V_T + V_s\lambda)}{V_s(V_s + V_T)(1-\lambda)} - \frac{(V_T + V_s\lambda)^{N_n-1}}{V_s(V_s + V_T)(1-\lambda)} (R_0(1-\lambda) + r(1+\lambda)). \quad (78)$$

In the last inward advancement toward the center of the evader region, the sweepers advance inwards and place the lower tips of their sensors at the center of the evader region. The time it takes the sweepers to complete this inwards motion is given by

$$T_{\text{in,last}}(n) = \frac{R_N}{V_s}. \quad (79)$$

R_N is calculated by similar steps as the calculation in Appendix B. Recalling that $\hat{R}_N = R_N - r$, we have that

$$\hat{R}_N = \frac{c_1}{1-c_2} + c_2^{N_n} \left(\tilde{R}_0 - \frac{c_1}{1-c_2} \right). \quad (80)$$

Substituting the coefficients in (80) yields

$$R_N = -\frac{2r}{1-\lambda} + c_2^{N_n} \left(\frac{R_0(1-\lambda) + r(1+\lambda)}{1-\lambda} \right). \quad (81)$$

Substituting the expression for R_N in $T_{\text{inlast}}(n)$ given in (79) yields

$$T_{\text{inlast}}(n) = -\frac{2r}{V_s(1-\lambda)} + c_2^{N_n} \left(\frac{R_0(1-\lambda) + r(1+\lambda)}{V_s(1-\lambda)} \right). \quad (82)$$

Since the time to sweep around radius \tilde{R}_i is obtained by multiplying \tilde{R}_i by $\frac{\lambda-1}{V_T}$, when multiplying (70) by $\frac{\lambda-1}{V_T}$, we can construct a difference equation for the sweep times. This difference equation is given by

$$T_{i+1} = c_2 T_i + c_3. \quad (83)$$

The coefficient c_3 is given by

$$c_3 = \frac{-2rV_s(\lambda-1)}{(V_s+V_T)V_T}. \quad (84)$$

The total time of spiral sweeps until the evader region is reduced to a circle with a radius that is equal to or smaller than $2r$ follows the derivation in Appendix C and is given by

$$\tilde{T}_{\text{spiral}}(n) = \frac{T_0 - c_2 T_{N_n-1} + (N_n - 1)c_3}{1 - c_2} \quad (85)$$

where the time of the first sweep is given by

$$T_0 = \frac{(R_0 - r)(\lambda - 1)}{V_T}. \quad (86)$$

The time it takes to sweep the last cycle before the evader region is reduced to a circle with a radius less than or equal to $2r$ is computed in Appendix D, and is given by

$$T_{N_n-1} = \frac{c_3}{1 - c_2} + c_2^{N_n-1} \left(T_0 - \frac{c_3}{1 - c_2} \right). \quad (87)$$

Plugging in the appropriate coefficients yields

$$T_{N_n-1} = \frac{2r}{V_T} + \frac{c_2^{N_n-1}}{V_T} (R_0(\lambda-1) - r(1+\lambda)). \quad (88)$$

Substituting the derived coefficients into (85) yields

$$\begin{aligned} \tilde{T}_{\text{spiral}}(n) &= \frac{(r-R_0)(V_s+V_T)}{V_T V_s} - \frac{2r(V_T+V_s\lambda)}{V_T V_s(1-\lambda)} \\ &+ \frac{2r(N_n-1)}{V_T} - \left(\frac{V_T+V_s\lambda}{V_s+V_T} \right)^{N_n} \left(\frac{(V_s+V_T)(R_0(\lambda-1)-r(\lambda+1))}{V_T V_s(1-\lambda)} \right). \end{aligned} \quad (89)$$

After completing sweep number $N_n - 1$, the sweepers advance toward the center of the evader region until the lower tips of their sensors are located at the center of the evader region. Following this advance, the sweepers need to perform a circular sweep of radius r around the center of the evader region in order to complete the cleaning of the evader region. The sweepers can complete this last circular sweep only if their velocities are high enough so that during the circular motion, no evader escapes the sweepers. Since the critical velocity for a spiral sweep is lower than the critical velocity for a circular sweep, the sweepers need to perform the last circular after spiral sweep number $N_n - 1$ only if their velocities satisfy

$$2r \geq V_T T_{\text{last}} + V_T T_{\text{inlast}} + R_N. \quad (90)$$

Satisfying (90) means that no evader escapes the sweepers. Before the last sweep, the evader region is reduced to a circle of radius R_N that satisfies

$$0 < R_N \leq 2r. \quad (91)$$

An alternative way to represent R_N is, $R_N = r(2 - \varepsilon)$. Therefore, ε can be written as

$$\varepsilon = \frac{2r - R_N}{r}, \quad 0 \leq \varepsilon < 2. \quad (92)$$

The last circular sweep occurs after the sweepers advance toward the center of the evader region and place the lower tips of their sensors at the center of the evader region. The last sweep is therefore a circular sweep by an angle of $\frac{2\pi}{n}$ around a circle of radius r centered at the center of the evader region. The time it takes the sweepers to complete it is given by

$$T_{\text{last}}(n) = \frac{2\pi r}{nV_s}. \quad (93)$$

Therefore, in order to perform the last circular sweep directly after spiral sweep number $N_n - 1$, the inequality in (90) that can be written as

$$\varepsilon \geq \frac{2\pi V_T + nV_T(2 - \varepsilon)}{nV_s} \quad (94)$$

has to hold. Equation (94) implies that in order to perform the last circular sweep directly after spiral sweep number $N_n - 1$, V_s has to satisfy that

$$V_s \geq \frac{2\pi V_T + nV_T(2 - \varepsilon)}{n\varepsilon}. \quad (95)$$

Substituting ε in (95) with the expression for ε from (92) yields that in order to perform the last circular sweep directly after spiral sweep number $N_n - 1$, V_s has to satisfy that

$$V_s \geq \frac{2\pi r V_T + nV_T R_N}{n(2r - R_N)}. \quad (96)$$

Rearranging terms and denoting the smallest possible ε that satisfies (95) as ε_c yields that

$$\varepsilon_c \geq \frac{2V_T(\pi + n)}{n(V_s + V_T)}. \quad (97)$$

Therefore, if the evader region's radius after sweep number $N_n - 1$ satisfies $R_N \geq r(2 - \varepsilon_c)$ or alternatively

$$R_N \geq \frac{2rnV_s - 2V_T\pi r}{n(V_s + V_T)}. \quad (98)$$

Then, the sweepers velocity is not sufficient to guarantee escape from the evader region. Demanding R_N satisfies (98) is equivalent to demanding that if V_s is not high enough and does not satisfy the inequality in (96), then the sweepers velocity is not sufficient to guarantee escape. In case these inequalities are violated, the sweepers perform another spiral sweep, which starts when the lower tips of their sensors are located at the center of the evader region. This spiral sweep starts when the center of each sweeper is at a distance of r from the center of the region and the time it takes to complete it is denoted by $T_l(n)$. It is

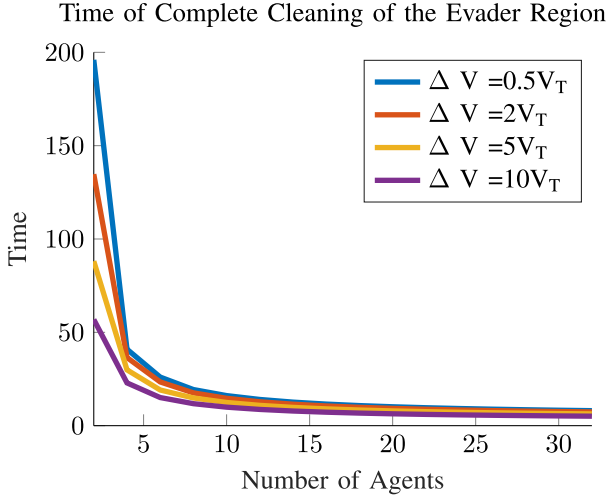


Fig. 11. Complete cleaning time of the evader region. We simulated sweep processes with an even number of agents, ranging from 2 to 32 agents that employ the spiral sweep process. We show results obtained for different values of velocities above the spiral critical velocity, i.e., different choices for ΔV . The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

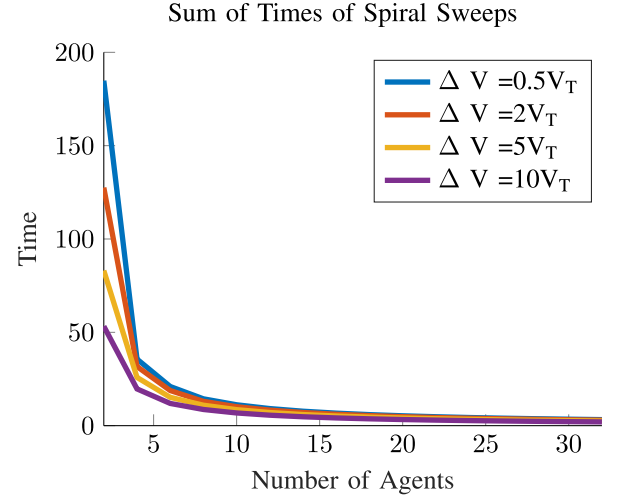


Fig. 12. Sum of spiral sweep times until complete cleaning of the evader region. We simulated spiral sweep processes with an even number of agents, ranging from 2 to 32 agents, which employ the multiagent spiral sweep process. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

given by

$$T_l(n) = \frac{r(\lambda - 1)}{V_T}. \quad (99)$$

We therefore introduce a characteristic function η that takes the values of 1 and 0. If the additional spiral sweep needs to be performed, $\eta = 1$ and, therefore, $T_l(n)$ is added to the sweep time. If no additional spiral sweep is required, $\eta = 0$. Therefore, the general term for $T_{\text{spiral}}(n)$ is given by

$$T_{\text{spiral}}(n) = \tilde{T}_{\text{spiral}}(n) + T_{\text{last}}(n) + \eta T_l(n). \quad (100)$$

$T_{\text{in}}(n)$ is given by the sum

$$T_{\text{in}}(n) = \tilde{T}_{\text{in}}(n) + T_{\text{in,last}}(n) + \eta T_{\text{in}_f}(n). \quad (101)$$

Let us denote by $T_{\text{in}_f}(n)$ the sweepers inward advancement time that corresponds to the spread of possible evaders that originated at the center of the evader region at the beginning of the last spiral sweep and had time of $T_l(n)$ to spread from the center at a velocity of V_T . Therefore, $T_{\text{in}_f}(n)$ is given by $T_{\text{in}_f}(n) = \frac{T_l(n)V_T}{V_s}$. Therefore, the total time of inward advancements is given by

$$T_{\text{in}}(n) = \tilde{T}_{\text{in}}(n) + \frac{R_N}{V_s} + \frac{\eta r(\lambda - 1)}{V_s}. \quad (102)$$

Substituting the terms in (100) yields

$$T_{\text{spiral}}(n) = \tilde{T}_{\text{spiral}}(n) + \frac{2\pi r}{nV_s} + \frac{\eta r(\lambda - 1)}{V_T}. \quad (103)$$

Fig. 11 presents the complete evader region's cleaning times for different numbers of sweepers. Complete cleaning times are the sum of spiral sweep and inward advancement times. Fig. 12 shows only the spiral sweep times component and Fig. 13 shows only the inward advancement times component. In each plotted

Sum of Times of Inward Advancements

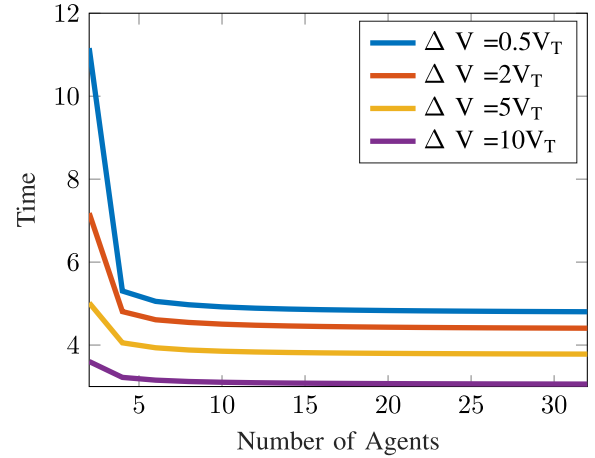


Fig. 13. Sum of inward advancement times until complete cleaning of the evader region. We simulated spiral sweep processes with an even number of agents, ranging from 2 to 32 agents. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

curve, the velocity of the sweepers is equal, regardless to the number of sweepers performing the search, in order to make a fair comparison between cleaning times of swarms with different number of sweepers. ΔV is taken with respect to the critical velocity of two sweepers, since as the number of sweepers increases, the critical velocity that enables them to complete the search decreases. Therefore, swarms with increasing number of sweepers are able to complete the search when moving at a velocity that is above the critical velocity of two sweepers while the converse does not necessarily hold. The critical velocity for the sweeper swarm is obtained by solving numerically the equation presented in Corollary 1 and, therefore, accounts for the spread of evaders during the sweeper's advancement toward the center of the evader region, hence guaranteeing no evader escapes. Fig. 14 shows that the gain in search time reduction

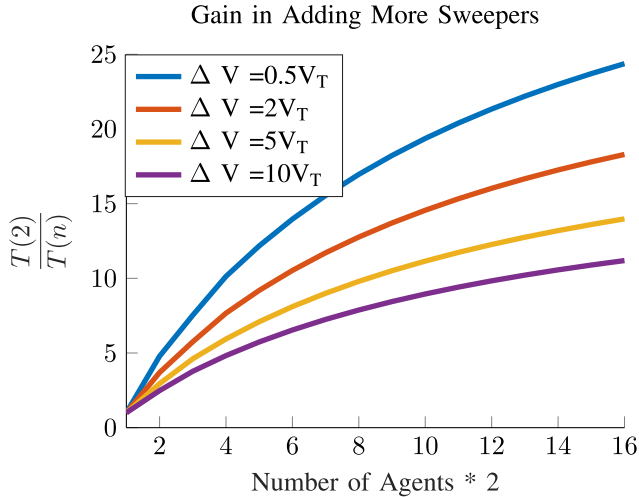


Fig. 14. Cleaning time gain obtained by adding more sweepers. We simulated sweep processes with an even number of agents, denoted by n , ranging from 2 to 32 agents that employ the spiral sweep process. In each curve, every point is obtained by the ratio between the sweep times of a two agent swarm and an n agent sweeper swarm. We show results obtained for different values of velocities above the spiral critical velocity, i.e., different choices for ΔV . The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

is more pronounced when velocities are closer to the critical velocity.

VI. COMPARATIVE ANALYSIS OF THE PROPOSED SEARCH STRATEGIES

This section provides a comparison between the obtained results for the circular and spiral sweep processes that were developed in the previous sections. In order to make a fair comparison between the total sweep times of sweeper swarms that can perform both the circular and spiral sweep processes, the number of sweepers and sweeper velocity must be the same in each of the tested spiral and circular swarms. The critical velocity required for sweepers performing the circular sweep process is higher than the minimal critical velocity required for sweepers performing the spiral sweep process. Therefore, in Fig. 15, we show the spiral sweep process's sweep times that are obtained for different values of velocities above the circular critical velocity. This means that the values of ΔV that are mentioned in the plots correspond to sweeper velocities that are almost twice the spiral critical velocities. Requiring a higher critical velocity implies that there are entire regions of operation where an evader area with a given radius can be cleaned by a swarm that performs the spiral sweep process but cannot be cleaned by a swarm that performs the circular sweep process. Fig. 16 compares the cleaning times of circular sweeping swarms and spiral sweeping swarms. The results are computed with the same agent velocities for both the circular and spiral multiagent sweep processes. The reduction in complete sweeping times that are achieved when sweepers employ the spiral search process are clearly observable. This result is independent of the number of the sweepers that perform the search or the velocity in which they move.

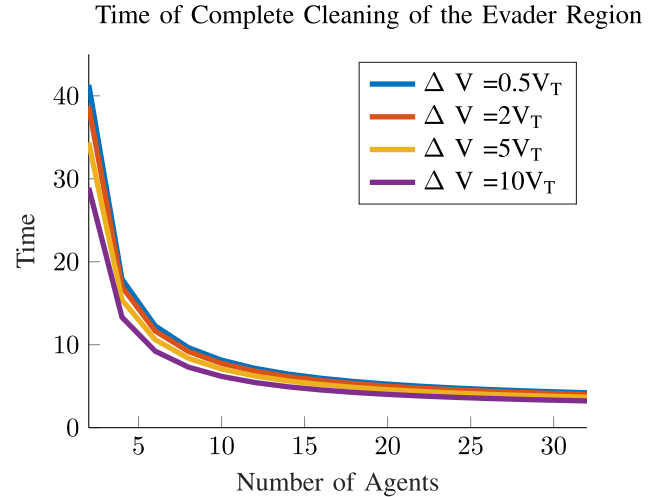


Fig. 15. Complete cleaning time of the evader region. We simulated sweep processes with an even number of agents, ranging from 2 to 32 agents, which employ the multiagent spiral sweep process. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$. The values of ΔV are above the critical velocity of the circular sweep process.

Time of Complete Cleaning of the Evader Region

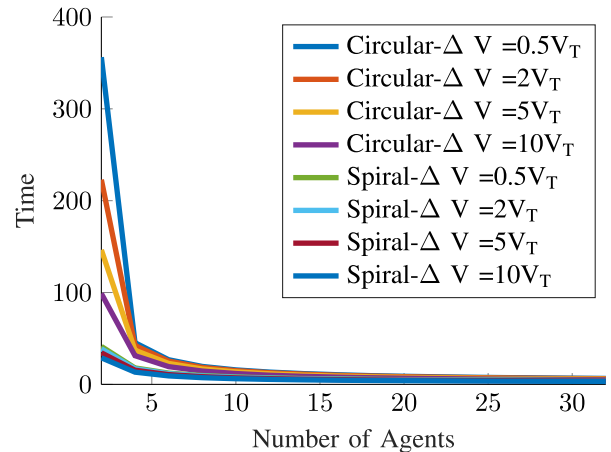


Fig. 16. Total search times until complete cleaning of the evader region for the circular and spiral sweep processes. We simulated sweep processes with an even number of agents, ranging from 2 to 32 agents, which employ the multiagent circular and spiral sweep processes. We show results obtained for different values of velocities above the circular critical velocity. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

VII. QUANTITATIVE COMPARISON OF PINCER-BASED AND SAME-DIRECTION SEARCH STRATEGIES

The purpose of this section is to compare between the results of pincer-based sweep processes developed in this article and the circular and spiral same-direction sweep processes in [29].

The critical velocity required for sweepers that perform the same-direction circular or spiral sweep processes is higher than the minimal critical velocity of their pincer sweep counterparts. Therefore, in order to compare same-direction and pincer circular or spiral strategies, all sweepers in the swarm move at velocities above the critical velocity of two sweepers performing the corresponding same-direction sweep. The critical velocity

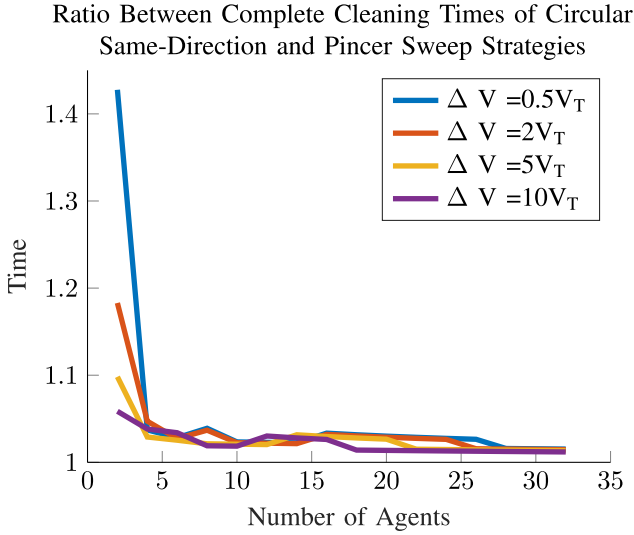


Fig. 17. Ratio between total search times until complete cleaning of the evader region for circular same-direction and pincer sweep processes. We simulated sweep processes with an even number of agents, ranging from 2 to 32 agents, which employ the multiagent same-direction and pincer circular sweep processes. We show results obtained for different values of velocities above the same-direction circular critical velocity. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

for two sweepers is chosen since it is greater than the critical velocity of search processes performed with more sweepers.

Fig. 17 depicts the ratio between the time it takes swarms employing same-direction circular sweeps and swarms performing their circular pincer sweep processes counterparts, to detect all evaders in the region. We conclude that for all choices of velocities above the same-direction circular critical velocity, the ratio is greater than 1, implying that same-direction circular sweeps require more time in order to clean the entire evader region.

Fig. 18 depicts the ratio between the time it takes swarms employing same-direction spiral sweeps to the time it takes swarms performing their spiral pincer sweep processes counterparts. When four or more sweepers perform the search process, for all choices of velocities above the same-direction spiral critical velocity, the ratio is greater than 1, implying that same-direction spiral sweeps require more time in order to clean the entire evader region. The only cases where same-direction spiral sweeps are better than their pincer-based counterparts are when sweepers move at velocities that are close to the critical velocity and where the sweep process is performed with exactly two sweepers. We observe that as the number of sweepers increases, the gain obtained from the cooperation between the sweeping pairs in pincer-based sweep processes decreases the sweeping time dramatically. When sweepers perform same-direction spiral sweeps, they have to sweep larger sections at each iteration in order to ensure no evader escapes, whereas in pincer-based spiral search strategies, sweeping these additional sections is unnecessary due to the complementary trajectories of the sweepers. A higher critical velocity implies that entire regions of operation where an evader region with a given radius can be searched by a swarm performing the same-direction spiral sweep process but

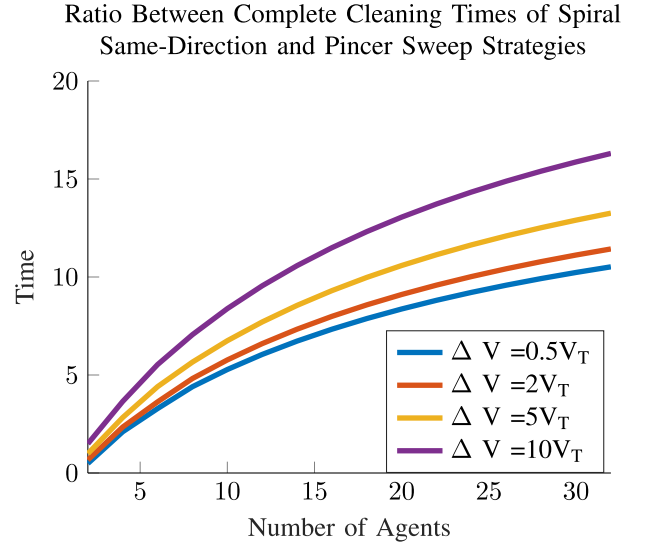


Fig. 18. Ratio between total search times until complete cleaning of the evader region for spiral same-direction and pincer sweep processes. We simulated sweep processes with an even number of agents, ranging from 2 to 32 agents, which employ the multiagent same-direction and pincer spiral sweep processes. We show results obtained for different values of velocities above the same-direction spiral critical velocity. The chosen values of the parameters are $r = 10$, $V_T = 1$, and $R_0 = 100$.

cannot be cleaned by a swarm performing the same-direction circular sweep process. This also implies that swarms performing pincer movement search strategies can sweep larger regions than their same-direction spiral counterparts.

VIII. CONCLUSION

This research considered a scenario in which a multiagent swarm of agents searched an area containing smart mobile evaders that were to be detected. There can be many evaders in this area, and potential evaders can be located at any point in an initial circular region of radius R_0 . The information the agents perceived only came from their own sensors, and evaders that intersected a sweeper's linear field of view were detected. Every agent in the swarm has a line-shaped sensor of length $2r$. All sweepers move with a speed of V_s and evaders move at a maximal speed of V_T . The sweepers objective was to "clean" or to detect all evaders that can move freely in all directions from their initial locations. The search time depends on the type of sweeping movement the sweeper swarm employs. The detection of evaders was done using predetermined and deterministic sweeping methods around the region, which result in the complete cleaning of the evader region, by shrinking the possible area in which evaders can reside to zero, in finite time.

In this article, we provided an analysis of the proposed sweep processes performance in terms of completion times of the search processes. We also provided a global balance of covered areas argument to obtain a lower bound on a searcher velocity that was independent of the search process. A critical velocity that depends on a novel proposed circular search pattern for a multiagent swarm ensuring satisfaction of the confinement task was then derived. When comparing this velocity to the lower bound on the critical velocity, results showed that it equals twice

the lower bound and, hence, was not optimal. Next, another critical velocity that depends on a novel pincer movement based spiral search process for a multiagent swarm that ensures the satisfaction of the confinement task was derived. We showed that the developed spiral critical velocity approaches the theoretical optimal critical velocity. No sweeping process can successfully detect all potential evaders in the region if sweepers move at a velocity below this lower bound. Therefore, both the developed circular critical velocities and the spiral critical velocities were compared to this lower bound. Afterward, we provided a comparison between the different search methods in terms of completion times of the sweep processes. Both total cleaning times and minimal searcher velocity required for a successful search were compared. At last, we provided a quantitative comparison between pincer-based and same-direction strategies.

A possible extension to this work is to replace the assumption that whenever an evader is in the field of view of the searcher's sensor, it is detected with probability 1. A more realistic model would be one based on the statistical properties of the actual detector. Another extension could be the introduction of a tuneable robustness parameter that compensates for an agent's inability to move in a constant speed throughout the entire search. Such a robustness parameter allows a margin between the evader region boundaries and the tip of the agent's sensor. This margin comes at the expense of permitting the agents to advance a lesser distance into the evader region after each iteration. Such a method can compensate for the simplified model where agents change their directions of travel instantaneously in the planar sweep scenario, thus providing a more realistic model that relies on the conducted research.

APPENDIX A

The number of sweep iterations that are required to reduce the evader region to be bounded by a circle with a radius that is less or equal to R_N is calculated in the following manner. We have that $R_{i+1} = c_2 R_i + c_1$. Therefore

$$R_N = c_2^N R_0 + c_1 \sum_{i=0}^{N-1} c_2^i = c_2^N \left(R_0 - \frac{c_1}{1-c_2} \right) + \frac{c_1}{1-c_2}. \quad (104)$$

Rearranging terms results in

$$\frac{R_N - \frac{c_1}{1-c_2}}{R_0 - \frac{c_1}{1-c_2}} = c_2^N. \quad (105)$$

Applying the natural logarithm function to (105) yields

$$\ln \left(\frac{R_N - \frac{c_1}{1-c_2}}{R_0 - \frac{c_1}{1-c_2}} \right) = N \ln c_2. \quad (106)$$

Therefore, the general form for the number of iterations N , it takes a sweeper swarm to reduce the evader region to be bounded by a circle with a radius requiring one final sweep before completely cleaning the evader region is given by

$$N = \left\lceil \frac{1}{\ln c_2} \ln \left(\frac{R_N - \frac{c_1}{1-c_2}}{R_0 - \frac{c_1}{1-c_2}} \right) \right\rceil. \quad (107)$$

APPENDIX B

The number of sweep iterations that are required to reduce the evader region to be bounded by a circle with a radius that is less than or equal to R_{N-2} is $N-2$. R_{N-2} is calculated as follows. We have that $R_{i+1} = c_2 R_i + c_1$. Therefore

$$\begin{aligned} R_{N-2} &= c_2^{N-2} R_0 + c_1 \sum_{i=0}^{N-3} c_2^i \\ &= c_2^{N-2} \left(R_0 - \frac{c_1}{1-c_2} \right) + \frac{c_1}{1-c_2}. \end{aligned} \quad (108)$$

Rearranging terms results in

$$\frac{R_{N-2} - \frac{c_1}{1-c_2}}{R_0 - \frac{c_1}{1-c_2}} = c_2^{N-2}. \quad (109)$$

Therefore, R_{N-2} is given by

$$R_{N-2} = \frac{c_1}{1-c_2} + c_2^{N-2} \left(R_0 - \frac{c_1}{1-c_2} \right). \quad (110)$$

APPENDIX C

The time it takes a multiagent swarm that performs either the circular or spiral sweep processes to reduce the evader region to its final radius before the completion of the sweep process is calculated as follows. The recursive relation between the next and current radius of the circle that bounds the evader region is given by

$$R_{i+1} = c_2 R_i + c_1. \quad (111)$$

Suppose that there exists a constant γ such that

$$\gamma R_i = T_i. \quad (112)$$

Therefore, multiplying (111) by γ yields

$$T_{i+1} = c_2 T_i + c_3 \quad (113)$$

where c_3 is given by $c_3 = \gamma c_1$. The time it takes to complete the first cycle around the evader region is $T_0 = \gamma R_0$. The time it takes to complete the last cycle before the evader region is bounded by a circle of radius r (for the circular sweep) or by a circle of radius $2r$ (for the spiral sweep) is the time it takes to complete a sweep when the evader region is bounded by a circle with a radius greater than R_{N-1} . This time is given by $T_{N-1} = \gamma R_{N-1}$. Summing over all cycle times except the initial one, by summing the cycle times given in (113) yields

$$\sum_{i=1}^{N-1} T_i = c_2 \sum_{i=1}^{N-1} T_i + c_2 (T_0 - T_{N-1}) + (N-1) c_3. \quad (114)$$

Rearranging terms results in

$$\sum_{i=1}^{N-1} T_i = \frac{c_2 (T_0 - T_{N-1}) + (N-1) c_3}{1-c_2}. \quad (115)$$

Since the total time it takes the swarm to clean the evader region includes also the time of the first sweep, T_0 is added to the summation as well. Thus, the total time it takes the swarm to reduce the evader region to be bounded by a circle with a radius less than or equal to r (for the circular sweep process) or $2r$ (for

the spiral sweep process) is given by

$$T = \sum_{i=0}^{N-1} T_i = \frac{T_0 - c_2 T_{N-1} + (N-1)c_3}{1 - c_2}. \quad (116)$$

APPENDIX D

The time it takes to complete a sweep around an evader region bounded by a circle with a radius of R_{N-1} is calculated as follows. From Appendix C, the recursive relation between the time it takes an agent to complete sweep number i and the time it takes to complete sweep number $i+1$ is given by

$$T_{i+1} = c_2 T_i + c_3. \quad (117)$$

Therefore

$$\begin{aligned} T_{N-1} &= c_2^{N-1} T_0 + c_3 \sum_{i=0}^{N-2} c_2^i \\ &= c_2^{N-1} \left(T_0 - \frac{c_3}{1-c_2} \right) + \frac{c_3}{1-c_2}. \end{aligned} \quad (118)$$

Rearranging terms results in

$$\frac{T_{N-1} - \frac{c_3}{1-c_2}}{T_0 - \frac{c_3}{1-c_2}} = c_2^{N-1}. \quad (119)$$

Therefore, the time it takes to complete a sweep around the evader region that is bounded by a circle with a radius of R_{N-1} is given by

$$T_{N-1} = \frac{c_3}{1-c_2} + c_2^{N-1} \left(T_0 - \frac{c_3}{1-c_2} \right). \quad (120)$$

APPENDIX E

The recursive relation between the next and current radius of the circle that bounds the evader region is given by

$$R_{i+1} = c_2 R_i + c_1. \quad (121)$$

Summing over the evader region radii up to the $N_n - 2$ cycle except the initial radius of the evader region is calculated by summing the radii given in (121). This results in

$$\sum_{i=1}^{N_n-2} R_i = c_2 \sum_{i=1}^{N_n-2} R_i + c_2(R_0 - R_{N_n-2}) + (N_n - 2)c_1. \quad (122)$$

Rearranging terms results in

$$\sum_{i=1}^{N_n-2} R_i = \frac{c_2(R_0 - R_{N_n-2}) + (N_n - 2)c_1}{1 - c_2}. \quad (123)$$

Since the sum of radii in (123) does not include the initial radius of the evader region R_0 is added to the summation as well. Thus, the desired sum of radii is given by

$$\sum_{i=0}^{N_n-2} R_i = \frac{R_0 - c_2 R_{N_n-2} + (N_n - 2)c_1}{1 - c_2}. \quad (124)$$

APPENDIX F

In this appendix, the time of inward advancements until the evader region is bounded by a circle with a radius that is

smaller or equal to r is computed. This time is denoted by $\tilde{T}_{in}(n) = \sum_{i=0}^{N_n-2} T_{in,i}$. This proof continues the derivation from Section V. After rearranging terms, (29) resolves to

$$\tilde{T}_{in}(n) = \sum_{i=0}^{N_n-2} T_{in,i} = \frac{(N_n - 1)r}{V_s + V_T} - \frac{2\pi V_T \sum_{i=0}^{N_n-2} R_i}{nV_s(V_s + V_T)}. \quad (125)$$

The term $\sum_{i=0}^{N_n-2} R_i$ is calculated in Appendix E. It is given by

$$\sum_{i=0}^{N_n-2} R_i = \frac{R_0 - c_2 R_{N_n-2} + (N_n - 2)c_1}{1 - c_2}. \quad (126)$$

R_{N_n-2} is calculated in Appendix B and is given by

$$R_{N_n-2} = \frac{c_1}{1 - c_2} + c_2^{N_n-2} \left(R_0 - \frac{c_1}{1 - c_2} \right). \quad (127)$$

Substituting the coefficients in (127) yields

$$R_{N_n-2} = \frac{nrV_s}{2\pi V_T} + \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n-2} \left(\frac{2\pi R_0 V_T - nrV_s}{2\pi V_T} \right). \quad (128)$$

Substituting the coefficients in (126) yields

$$\begin{aligned} \sum_{i=0}^{N_n-2} R_i &= -\frac{R_0 n(V_s + V_T)}{2\pi V_T} + \frac{n^2 r V_s (V_s + V_T)}{(2\pi V_T)^2} \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right) \\ &+ \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n-1} \left(\frac{n(V_s + V_T)(2\pi R_0 V_T - nrV_s)}{(2\pi V_T)^2} \right) \\ &+ \frac{rnV_s(N_n-2)}{2\pi V_T}. \end{aligned} \quad (129)$$

Substituting $\sum_{i=0}^{N_n-2} R_i$ from (129) into (125) results in

$$\begin{aligned} \tilde{T}_{in}(n) &= \sum_{i=0}^{N_n-2} T_{in,i} \\ &= \frac{R_0}{V_s} - \frac{nr}{2\pi V_T} - \left(1 + \frac{2\pi V_T}{n(V_s + V_T)} \right)^{N_n-1} \left(\frac{2\pi R_0 V_T - nrV_s}{2\pi V_T V_s} \right). \end{aligned} \quad (130)$$

APPENDIX G

For a swarm performing the spiral sweep process, the inward advancement times until the evader region is reduced to a circle with a radius that is smaller or equal to $2r$ are denoted by $\tilde{T}_{in}(n) = \sum_{i=0}^{N_n-2} T_{in,i}$. This proof continues the derivation from Section VI. After rearranging terms, (77) resolves to

$$\sum_{i=0}^{N_n-2} T_{in,i} = \frac{2r(N_n - 1) - (\lambda - 1) \sum_{i=0}^{N_n-2} \tilde{R}_i}{V_s + V_T}. \quad (131)$$

The term $\sum_{i=0}^{N_n-2} \tilde{R}_i$ is calculated in Appendix E. It is given by

$$\sum_{i=0}^{N_n-2} \tilde{R}_i = \frac{\tilde{R}_0 - c_2 \tilde{R}_{N_n-2} + (N_n - 2)c_1}{1 - c_2}. \quad (132)$$

\tilde{R}_{N_n-2} is calculated in Appendix B and is given by

$$\tilde{R}_{N_n-2} = \frac{c_1}{1 - c_2} + c_2^{N_n-2} \left(\tilde{R}_0 - \frac{c_1}{1 - c_2} \right). \quad (133)$$

Substituting the coefficients in (133) yields

$$\tilde{R}_{N_n-2} = -\frac{2r}{1-\lambda} + c_2^{N_n-2} \left(\frac{R_0(1-\lambda) + r(1+\lambda)}{1-\lambda} \right). \quad (134)$$

Denoting by c_4 the expression

$$c_4 = \frac{V_s+V_T}{V_s(1-\lambda)} \left(\frac{V_T+V_s\lambda}{V_s+V_T} \right)^{N_n-1}. \quad (135)$$

Substituting the coefficients in (132) yields

$$\sum_{i=0}^{N_n-2} \tilde{R}_i = \frac{(R_0-r)(V_s+V_T)}{V_s(1-\lambda)} + \frac{2r(V_T+V_s\lambda)}{V_s(1-\lambda)^2} - c_4 \left(\frac{R_0(1-\lambda)+r(1+\lambda)}{1-\lambda} \right) - \frac{2r(N_n-2)}{1-\lambda}. \quad (136)$$

Substituting $\sum_{i=0}^{N_n-2} \tilde{R}_i$ from (136) into (131) yields

$$\tilde{T}_{in}(n) = \frac{2r}{V_s+V_T} + \frac{R_0-r}{V_s} + \frac{2r(V_T+V_s\lambda)}{V_s(V_s+V_T)(1-\lambda)} - \frac{(V_T+V_s\lambda)^{N_n-1}}{V_s(V_s+V_T)(1-\lambda)} (R_0(1-\lambda) + r(1+\lambda)). \quad (137)$$

APPENDIX H

In this appendix, we prove that after the completion of a spiral sweep, the evader region is again circularly shaped. We prove this result for a two sweeper swarm. The extension to any number of even sweepers is straightforward. We denote by t_θ the time it takes a sweeper to sweep an angle of $\theta(t_\theta)$ around the evader region. t_θ is given by

$$t_\theta = \frac{(R_i - r) \left(\exp \left(\frac{\theta V_T}{\sqrt{V_s^2 - V_T^2}} \right) - 1 \right)}{V_T}. \quad (138)$$

Similarly, we denote by t_π the time it takes a sweeper to sweep an angle of $\theta(t_\pi)$ around the evader region. t_π is given by $t_\pi = \frac{(R_i-r)(\exp(\frac{\pi V_T}{\sqrt{V_s^2-V_T^2}})-1)}{V_T}$. The trajectory of the tip of the sensor that is closest to the center of the evader region of a sweeper that travels counter clockwise is given by

$$L_{CCW}(t_\theta) = (R_i - 2r + V_T t_\theta) [\sin(\theta(t_\theta)), \cos(\theta(t_\theta))]. \quad (139)$$

The trajectory of the sensor's tip of a clockwise travelling sweeper closest to the center of the evader region is given by

$$L_{CW}(t_\theta) = (R_i - 2r + V_T t_\theta) [\sin(2\pi - \theta(t_\theta)), \cos(2\pi - \theta(t_\theta))]. \quad (140)$$

At time t_θ , the counter clockwise sweeper detects evaders up to point $L_{CCW}(t_\theta)$. From t_θ to t_π , the evader region expands from $L_{CCW}(t_\theta)$ in all directions with a maximal speed of V_T for $t_\pi - t_\theta$, resulting in a spread of radius $V_T(t_\pi - t_\theta)$ in all directions. Hence, the wavefront from $L_{CCW}(t_\theta)$ is defined by the curve

$$E(\theta, \psi) = L_{CCW}(t_\theta) + V_T(t_\pi - t_\theta) [\sin(\psi), \cos(\psi)] \quad (141)$$

for all $\psi \in [0, 2\pi]$. This shows the expansion from $L_{CCW}(t_\theta)$ at time t_π . Evaders that spread the furthest distance from the center of the region are evaders that move along the ray between the region's center and the position where the lower tip of the sensor

sweeps at time t_θ . Combining this insight with (141) yields that at time t_π , these points satisfy that their distance from the center of the evader region is

$$R(t_\pi) = R_i - 2r + V_T t_\theta + V_T(t_\pi - t_\theta) = R_i - 2r + V_T t_\pi. \quad (142)$$

A calculation for the clockwise sweeping agent will result in the exact same distance of the furthest points from the center of the evader region that originated from the right half-plane spread of evaders. Therefore, after the completion of the sweep, the evader region will be a circle of radius $R_i - 2r + V_T t_\pi$.

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