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ON A MULTIPLE REGISTRATION PROBLEM

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SUMMARY

Suppose a planar shape S, colored in black on a white background, is sampled along several parallel lines, resulting in a set of black segments on the sampling lines. Denote by $L(\theta)$ the set of points in the plane that belong to the K parallel lines $\{l_j(\theta), j=1,2,...,K\}$ in the direction θ, i.e. $L(\theta) = \{(x,y)|(x,y) \in l_j(\theta), j=1,2,...,K\}$. The sampling process yields the set of points defined by $M(S,\theta) = S \cap L(\theta)$. If the shape is sampled along several groups of parallel lines in different directions, say θ_i, $i=1,2,...,P$, then the set $\bigcup_{i=1}^{P} M(S,\theta_i)$ provides information equivalent to sampling S on a grid determined by the union of the sets $L(\theta_i)$. If this grid is "dense" in the region where the object lies, this sampling provides a good evaluation of the shape S. Suppose, however that we are not given the sets $M(S,\theta_i)$ but rather some randomly translated versions of these, i.e. we only have access to $M^*(S,\theta_i) = \{M(S,\theta_i)+V_i\} = \{(x,y)+V_i|(x,y) \in M(S,\theta_i)\}$, where $\{V_i\}$ is a set of unknown vectors. Such a situation arises if the object makes random translatory moves between the samplings or if we do not know the relative

positions of the sets of parallel lines along which the object was sampled. The question that now arises is whether there exists a way to integrate the samples $M^*(S,\theta_i)$ so as to get good evaluations of the shape that generated them. Of course, ideally we would like to somehow determine the displacements V_i and then take the union of $\{M^*(S,\theta_i) - V_i\}$.

Let us place the sets $M^*(S,\theta_i)$ assuming that they were displaced by the vectors U_i. There is a simple way to check whether the assumed set of displacement vectors $\{U_i\}$ could be the true set $\{V_i\}$. Consider the grid generated by the union of lines $\bigcup_{i=1}^{P}\{L(\theta_i) - U_i\}$ and define the set of intersection points in this grid to be $C_G(\{U_i\})$. Let $C_B(\{U_i\})$ be the set of (black segment) intersection points in $\bigcup_{i=1}^{P}\{M^*(S,\theta_i) - U_i\}$. The vectors U_i have been incorrectly chosen if

$$\left[\bigcup_{i=1}^{P}\{M^*(S,\theta_i)-U_i\}\right]\cap C_G(\{U_i\})\neq C_B(\{U_i\})$$

since this implies the existence of points in the plane that, according to one of the assumed samples, say $\{M^*(S,\theta_i) - U_i\}$, belong to S, whereas according to some other sample they belong to the exterior of S. Therefore we know how to test a set of placement vectors $\{U_i\}$ for "correctness". If such a set of placement vectors is not ruled out as incorrect by the preceding test, then there exists a shape S that could have produced the measurements $\bigcup_{i=1}^{P}\{M^*(S,\theta_i)-U_i\}$ and we evaluate that shape by looking at the set of (suitably placed) measurements.

The problem that remains to be solved is the following: Given the measurements

$M^*(S,\theta_i)$, determine sets of vectors that yield legal placements of the measurements in the plane, according to the above discussed rule. We can choose $U_1 = 0$, since only the relative positions of the measurements is important. It is also clear that, for the overall placement to be correct, it has to yield correct relative placements of each pair of measurements. So $U_j - U_i$ has to belong to the range of correct placements of the j'th measurement w.r.t. the i'th one. We shall denote these "binary" ranges of correct placements by $R\{j|i\}$ (obviously $R\{j|i\} = -R\{i|j\}$). Therefore we have the following procedure for determining a "correct" overall placement $\{U_i\}$.

A. For each pair of measurements $\{M^*(S,\theta_i),M^*(S,\theta_j)\}$ determine the legal placement range as the set of translation vectors that yield correct binary placements according to the "intersection" criterion.

B. Determine a set of vectors $U_1 = 0, U_2, ..., U_P$, so as to have $U_j - U_i \in R\{j|i\}$ for all pairs i,j. Determining the placements $\{U_i\}$ according to this rule is a *multiple registration problem.*

In our case it is easy to see that the binary legal placement ranges $R\{j|i\}$ are collections of parallelogram-shaped areas. In general, if for all i and j, $R\{j|i\}$ is the interior of a convex polygon then the registration problem, i.e. the determination of a feasible placement, can be solved via linear programming. For placement ranges that are collections of convex polygonal shapes we have to solve a linear programming problem for each combination of simple convex shapes, to determine feasible solutions. If the basic ranges are discrete sets of vectors, the determination of a feasible solution can only be done by exhaustive search, the problem being NP-complete (reduction to partitioning).

We note that it would be interesting to find an algorithm that, given $R\{j|i\}$, determines the regions $\tilde{R}_i$ for i=2,3,..,P, having the property that for *any* $U_i^* \in \tilde{R}_i$ there exists a feasible solution of the registration problem having $U_i = U_i^*$. The regions $\tilde{R}_i$ measure the total uncertainity about V_i, given the randomly translated sampling data. Also, it would be interesting to find efficient solutions for the registration problem in other particular cases of placement ranges $R\{j|i\}$, like general convex regions (circles, ellipses, etc).

The issues discussed in this note arose in our investigation on determining the shape of moving objects from sparse samplings of edge-crossing measurements.

REFERENCE

M. Lindenbaum and A. Bruckstein, "Determining Object Shape from Local Velocity Measurements", EE. Report 599, Technion, IIT, Haifa, June 1986.

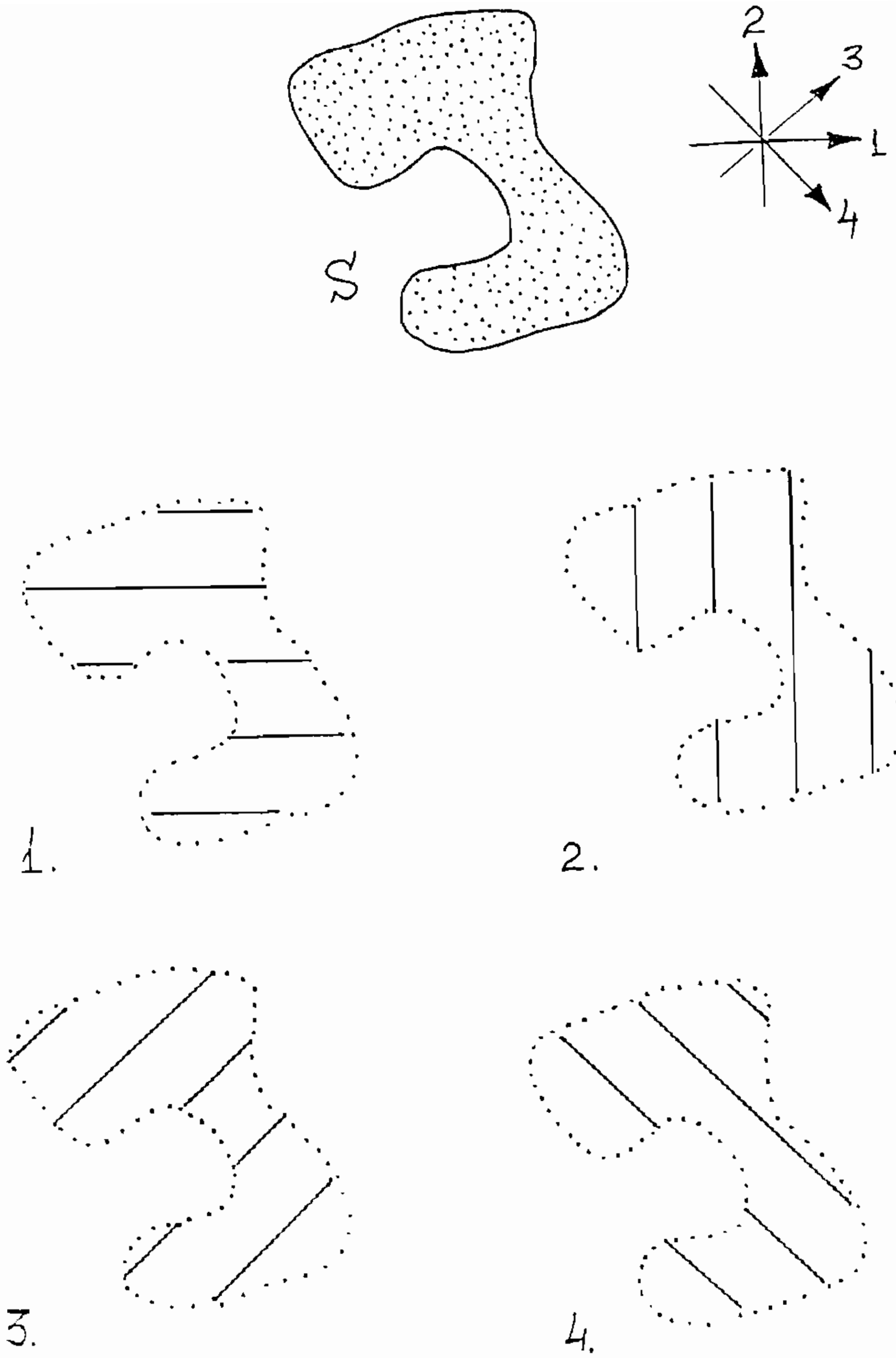
FIGURES

FIGURE 1: A planar shape and its samples in 4 directions.

FIGURE 2: Ranges of correct "binary" placements $R\{j|i\}$.

FIGURE 3: A legal and an illegal placement for samples 1 and 2.

FIGURE 4: Complete legal registrations produced by linear programming.



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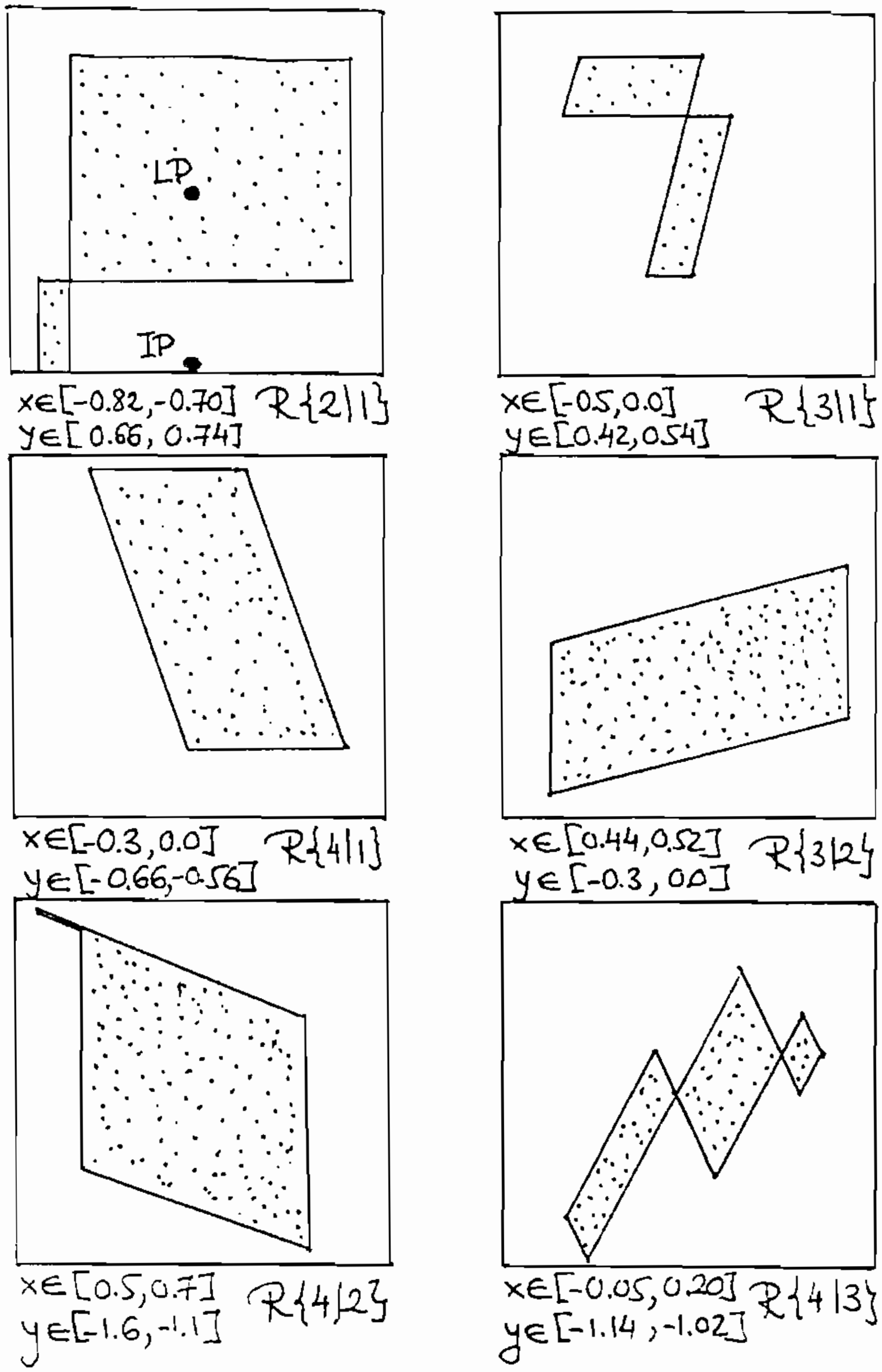
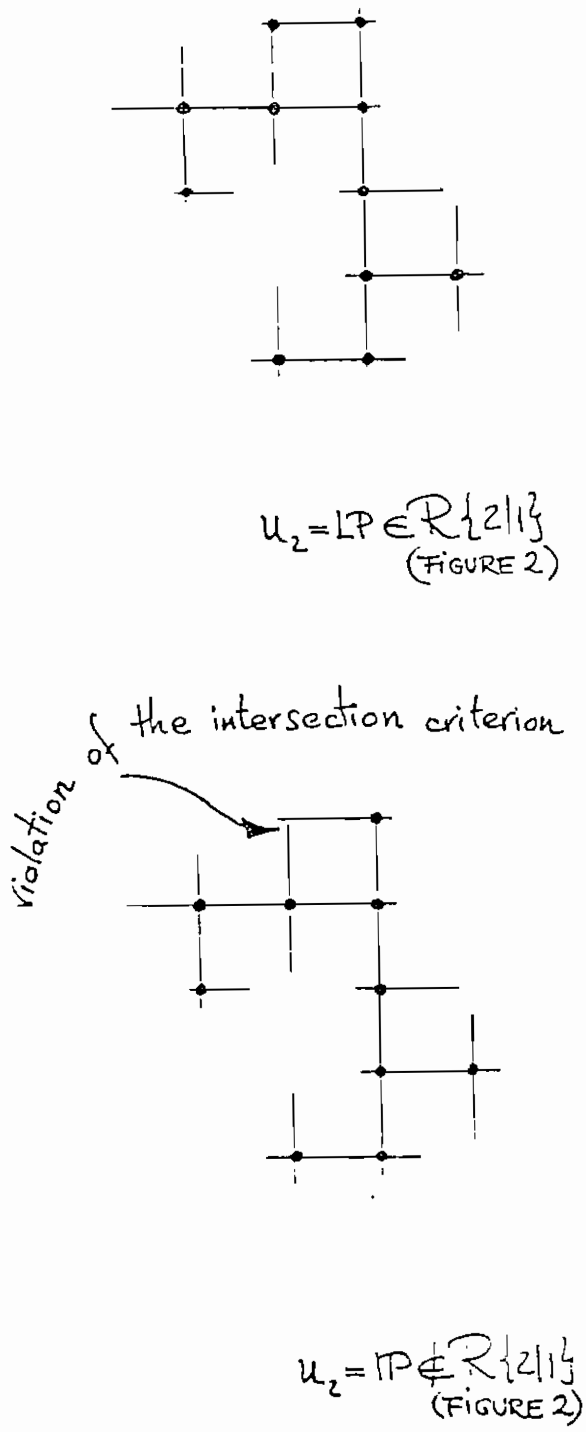


FIGURE 2: Ranges of correct "binary" placements $R\{j|i\}$.



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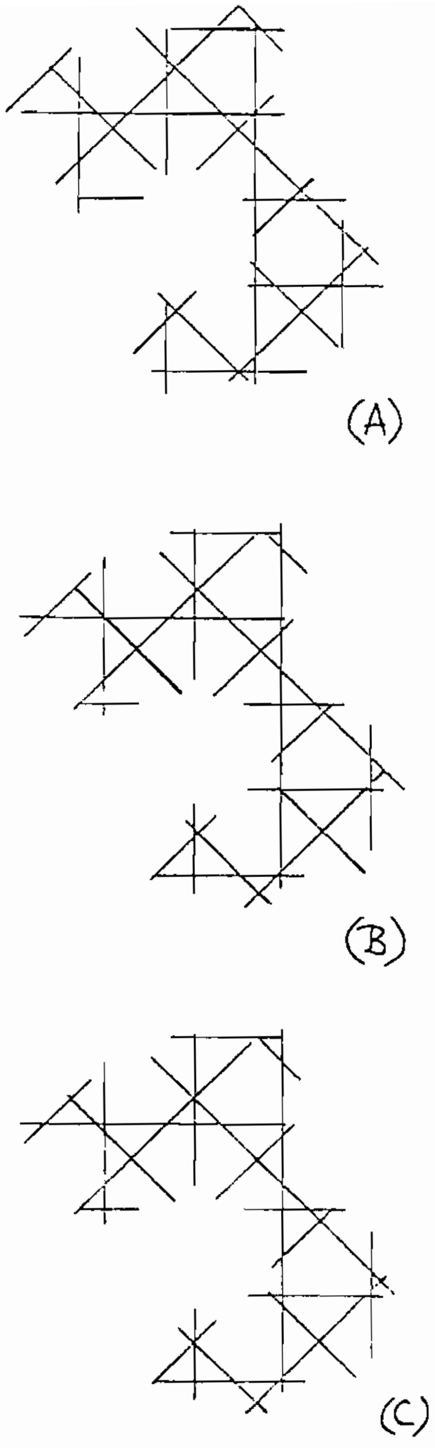

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