VARIATIONAL METHODS IN IMAGE PROCESSING

DO WE KNOW WHAT TO OPTIMIZE FOR?

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IN MEMORY OF

Professor Azriel Rosenfeld ז"ל

a pioneer of the picture processing field
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We shall therefore briefly discuss here two avenues where the aims for the image analysis & processing were to generate <u>rerenderings</u> for better IMAGE VISUALIZATION (?).

I shall argue that in many cases ART is better at this and that we may try to imitate ~~ART~~ via our (variational/PDE) bag of tools.

... but IMAGES are in fact 2D arrays of numbers, i.e. 2Dimensional data and one can and should present this data to the eyes in ways that will enhance the viewer's UNDERSTANDING. (PLEASURE!)

In fact: the entire field of

IMAGE PROCESSING

is about producing new images from old ones, so as to enhance their value to the human / or computerized (machine) OBSERVER. /

IMAGES

are meant to be seen, they are the results of imaging processes and imaging processes are attempts to do visualization

"IMAGE VISUALIZATION"

therefore should be considered a TAUTOLOGY: Visualize the results of a visualization process! It seems all we have to do is show/display the image!

. . .

CASE STUDIES in IMAGE VISUALIZATION

- RERENDERING IMAGES
  - Halftoning
  - Requantization
  - Color Palette modifications

my example: DigiDÜRER

- LOCATING & INTEGRATING EDGES
  - Image Gradient Fields
  - Saliency Measures
  - Level Sets & Flows

my examples:
- M-H & H-C edge detectors revisited
- Image Flows & One-liners

LOCATION & INTEGRATION of EDGES

• in work with R. Kimmel and with V. Machervaks & Gill Barequet

we aimed to:

1) provide a variational (optimization) framework for edge integration

2) analyse the image via some natural "flows" that will implicitly do edge integration & generate

ONE LINER REPRESENTATIONS for IMAGES (gray level pictures!)

IMAGE FLOWS and
ONE LINERS

- **Image Flows:**

Associate to $I(x,y)$ the gradient vector field

$$[\nabla I(x,y)] = \left[\frac{\partial I}{\partial x} \;\; \frac{\partial I}{\partial y} \right]$$

and the gradient magnitude image

$$G(x,y) = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

From an initial point we would like "to flow" into an edge in the image, i.e. toward a ridge of local maxima of $G(x,y)$.

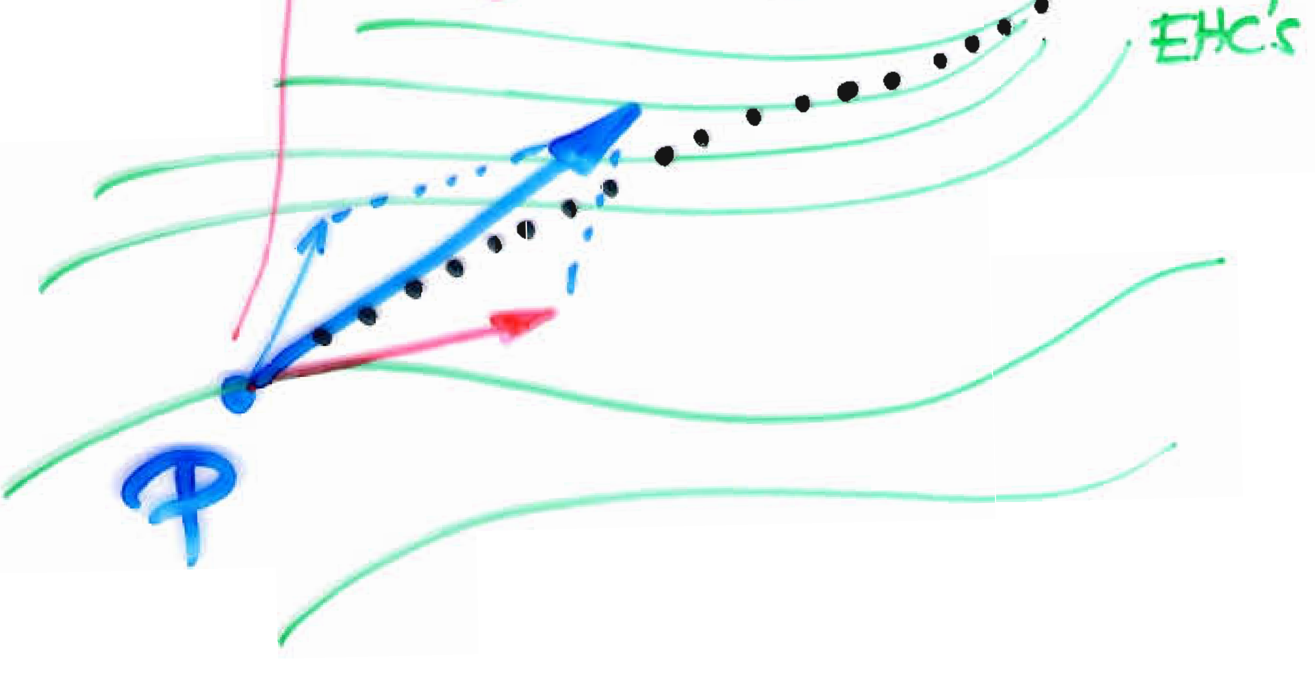
One possible edge exploration flow

is:

$$\underbrace{\frac{dP}{dt}}_{\text{velocity vector of } P} = \alpha \frac{(\nabla I)_\perp}{\|\nabla I\|} + (1-\alpha)\frac{\nabla G}{\|\nabla G\|}$$

directs flow along EHC's of I

makes P climb toward higher gradient magnitudes.



ALGORITHM

1) Do Edge Exploration from many Starting Points in Image Plane

2) Detect & Rank Most Important Edges - "the curves toward which many Edge Exploration Processes converge!"

3) Connect the Most Important Edges to form One liners.

PART 1:

CLASSICAL RESULTS ON
VARIATIONAL PRINCIPLES

(See e.g. Cornelius Lanczos
THE VARIATIONAL PRINCIPLES
of MECHANICS
dedicated to Einstein)

PART 2:

DO WE KNOW WHAT TO OPTIMIZE FOR?
or
the ART of applying VARIATIONAL METHODS

- on Laplacian Snakes
- on (over)parametrization and its benefits.

THE VARIATIONAL PRINCIPLES OF IMAGE ANALYSIS

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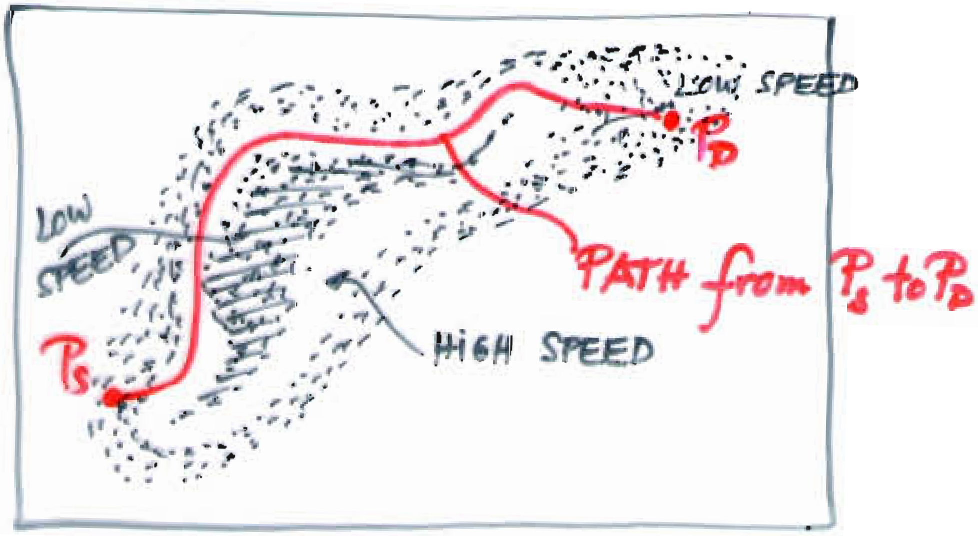
LEAST-TIME/COST TRAJECTORIES

(Fermat!)

In the (x,y) plane suppose that the speed of an agent is given by $(n(x,y))^{-1}$



$$\left|\frac{dP}{dt}\right| = (n(x,y))^{-1}$$

TOTAL TRAVEL TIME $\Delta T = \int n \, ds$ and if the path is parametrized by travel time this is

$$\Delta T = \int_{t_S}^{t_D} n(x,y)\sqrt{\dot{x}^2+\dot{y}^2}\,dt = \int_{t_S}^{t_D} 1\,dt = t_D - t_S$$

otherwise it is

$$\Delta T = \int_0^1 n(x,y)\sqrt{\dot{x}^2+\dot{y}^2}\,d\tau \quad \text{for } \tau \in [0,1].$$

In fact the functional

$$\int \underbrace{m(x(\tau),y(\tau))\sqrt{\dot{x}(\tau)^2+\dot{y}(\tau)^2}}_{F(x,y;x',y')}\,d\tau$$

is reparametrization invariant, as it should be!

THE BASIC QUESTION:

Given P_S (source) and P_D (destination) what is the path of shortest traversal time?

Basic CALCULUS of VARIATION yields:

EL equations $\frac{d}{d\tau}F_{x'} = F_x \quad \frac{d}{d\tau}F_{y'} = F_y$

or

$$\begin{cases} \frac{d}{d\tau}\, m(x,y)\frac{x'}{\sqrt{x'^2+y'^2}} = m_x(x,y)\sqrt{x'^2+y'^2} & (*) \\ \frac{d}{d\tau}\, m(x,y)\frac{y'}{\sqrt{x'^2+y'^2}} = m_y(x,y)\sqrt{\dot{x}'^2+y'^2} \end{cases}$$

$$(*)\ (m_x x'+m_y y')\frac{x'}{\sqrt{\cdots}} + m\,\frac{x''\sqrt{\cdots} - x'\,\frac{x'x''+y'y''}{\sqrt{\cdots}}}{(\sqrt{\cdots})^2} =$$

$$= m_x\sqrt{\cdots}$$