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Science: Mathematicians learn how to read the ant-trails

By IAN STEWART

'Go to the ant, thou sluggard; consider her ways, and be wise,' says Proverbs, vi, 6. Although the writer of this line was probably thinking of ants' legendary dedication to hard work, the ways of ants have a lot more to teach the seeker after wisdom – for instance, how to find the shortest route between two points without having a birds' eye view of the terrain. Now a mathematician in Israel, studying 'pursuit problems', has discovered how ants can achieve the feat by repeatedly applying simple local rules.

As a 'scout' ant searches for food, it leaves a faint trail of pheromones. Other ants that follow the trail lay down their own pheromones, making the trail stronger and straighter. Because individuals, such as the scout itself, move rather randomly, it is hard to see how the ants collectively 'know' the shortest path.

Many people, including the physicist Richard Feynman, have wondered at their ability. The consensus is that the straight path ants settle on is a result of a sequence of successive improvements. Although the route taken by the scout is random, the ants that follow modify the route and make it more efficient. There is evidence that ants that are forced to make sharp changes in direction either perform a U-turn, or keep going but lay down less pheromone. Computer simulations show that these simple rules for ant behaviour lead to a build-up of pheromone along tracks that are relatively straight (New Scientist, Science, 27 March).

Feynman's explanation was similar. He thought that ants trying to follow the original, random path tend to overshoot at corners and wander off randomly until they meet the path again. The net effect is to eliminate sharp bends and to cause pheromone to accumulate along straight-line paths. This explanation is reminiscent of the quantum explanation of why – in classical physics – light always takes the shortest path between any two points. In the quantum world, the light follows all possible paths, but when these are superposed, the wave-functions interfere destructively away from the classical path. Feynman pioneered this kind of explanation in physics, so it is not surprising that he developed a similar theory for ant trails: randomly moving ants superpose their pheromones, and the peak concentration is along the straight-line path.

Physical analogies and computer experiments are all very well, but mathematicians prefer proofs. Alfred Bruckstein of Technion University in Haifa, Israel, set up a rigorous mathematical model that implements a particular version of Feynman's suggestion. He proved that as successive ants follow each other, their paths become ever more straight (The Mathematical Intelligencer, Spring 1993, p 59).

Bruckstein's model replaces the pheromones by dynamical rules, but this does not deny the importance of pheromones in practice: the rules are a simplified version of 'follow the previous pheromone trail but smooth it out a bit'.

The model lies in the general area of 'pursuit problems', which date back to Leonardo da Vinci. A number of objects – usually represented by mathematical points – chase each other, so that at each instant each object is moving directly towards some other chosen object. For example, four beetles start from the corners of a square, and each one chases the next. They spiral in and meet at the centre in the same time that it would take one beetle to traverse the edge of the square. A basic pursuit theorem states that if several objects chase each other in sequence (so that A chases B chases C chases D chases A, say) then eventually they will all meet at the same point.

In Bruckstein's model, one scout ant sets off and lays down a trail. A second ant, moving at the same speed, starts some distance behind, but obeys the pursuit rule: 'Always head towards the scout.' A third ant pursues the second in the same manner, and so on. The motion of each ant is determined by a differential equation, which states that its velocity vector is always directed towards the current position of the previous ant.

Bruckstein considered the angle through which a given ant had turned at a given time, and estimated how this angle changes from one trip to the next. He found that the angle tended to zero, implying that the paths become ever more straight.

Bruckstein's theorem should be seen as a contribution to the mathematics of pursuit, motivated by ant trails, rather than a theory of ant behaviour. Real ants wander at varying speeds along a pheromone-splashed landscape, rather than playing follow-the-leader at uniform speeds. Indeed, it would be interesting to model the deposition of pheromones more directly: the analogy with quantum mechanics is quite strong and techniques similar to Feynman path integrals – which superpose contributions from all possible paths – might be useful.

Be that as it may, Bruckstein's theorem provides an elegant example of how systems that obey purely local rules can collectively settle into global optima. Many cases of collective animal behaviour, such as the flocking of birds or the movements of shoals of fish, display surprising large-scale patterns, giving the impression that the entire flock or shoal has some kind of overall 'consciousness' or sense of purpose. In fact, this appearance is an illusion – it is a collective response arising from the combination of simple rules followed by each individual. The important thing about the ant-trail pursuit problem is that for once we can pin down mathematically the precise manner in which the collective pattern is generated.