Design of Planar Shapes for Precise Registration

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PROBLEM: DESIGN FIDUCIALS THAT ENABLE SUBPIXEL IMAGE REGISTRATION

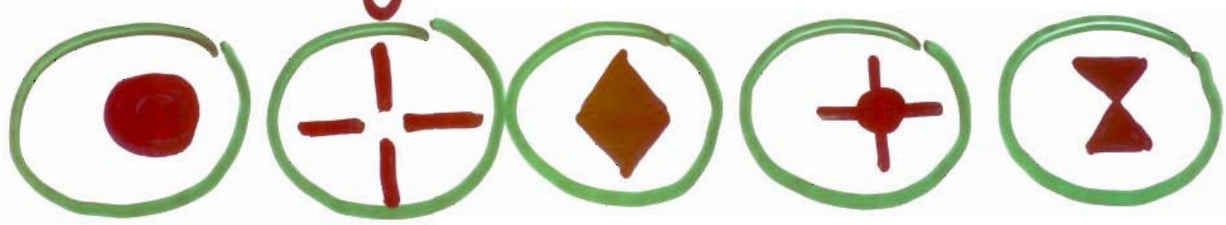
APPLICATIONS

automatic visual inspection
of Printed Circuit Boards or
VLSI Silicon wafers

location problems in robotic
assembly or soldering

overlay placements in
etching, printing etc... etc...

FIDUCIALS are widely used. They come in many SHAPES:



They are viewed by CAMERAS that SAMPLE & QUANTIZE THE IMAGE so

IT MAKES SENSE TO ASK, GIVEN SOME KNOWLEDGE OF THE CAMERA OPERATION, WHAT ARE THE SHAPES THAT YIELD BEST LOCATION ACCURACY.

THE MATHEMATICAL FORMALIZATION:

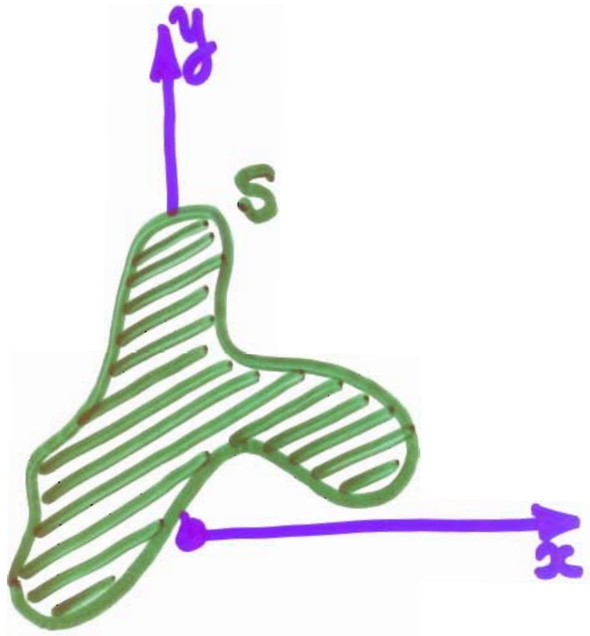
- FIDUCIAL IMAGE is BINARY (Bl/Wh) with shape described via

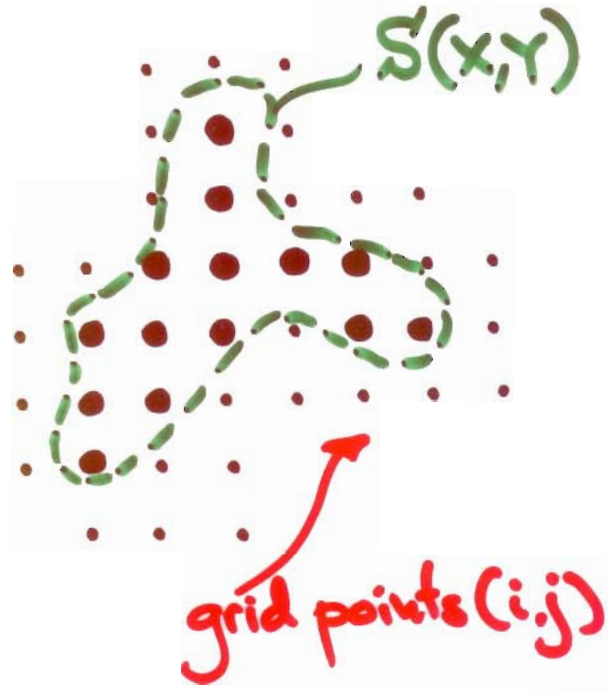
$$\chi_S(x,y) = \begin{cases} 1 & (x,y) \in S, \text{ ie } I(x,y) = Bl \\ 0 & (x,y) \notin S, \text{ ie } I(x,y) = Wh \end{cases}$$

- Translated versions of S, by (X,Y), are digitized by sampling on the UNIT GRID $\{(i,j) \in \mathbb{Z}^2\}$, yielding binary images

$$B(i,j) = \begin{cases} 1 & (i,j) \in S(X,Y) \\ 0 & (i,j) \notin S(X,Y) \end{cases} =$$

$$= \chi_{S(X,Y)}(i,j) = \chi_S(i-X, j-Y)$$





Suppose we are given the shape S (in its "master" coordinates) and the digitized image B(i,j) of a translated version of S

1) HOW SHOULD WE ESTIMATE THE TRANSLATION (X,Y) from B(i,j)?

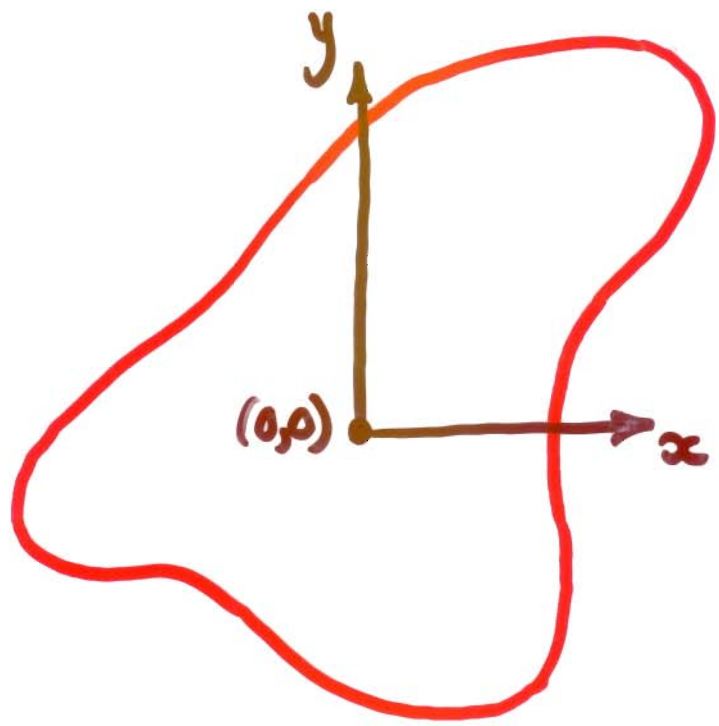
2) WHAT IS THE PRECISION IN ESTIMATING (X,Y), WHAT INFLENCES THIS PRECISION?

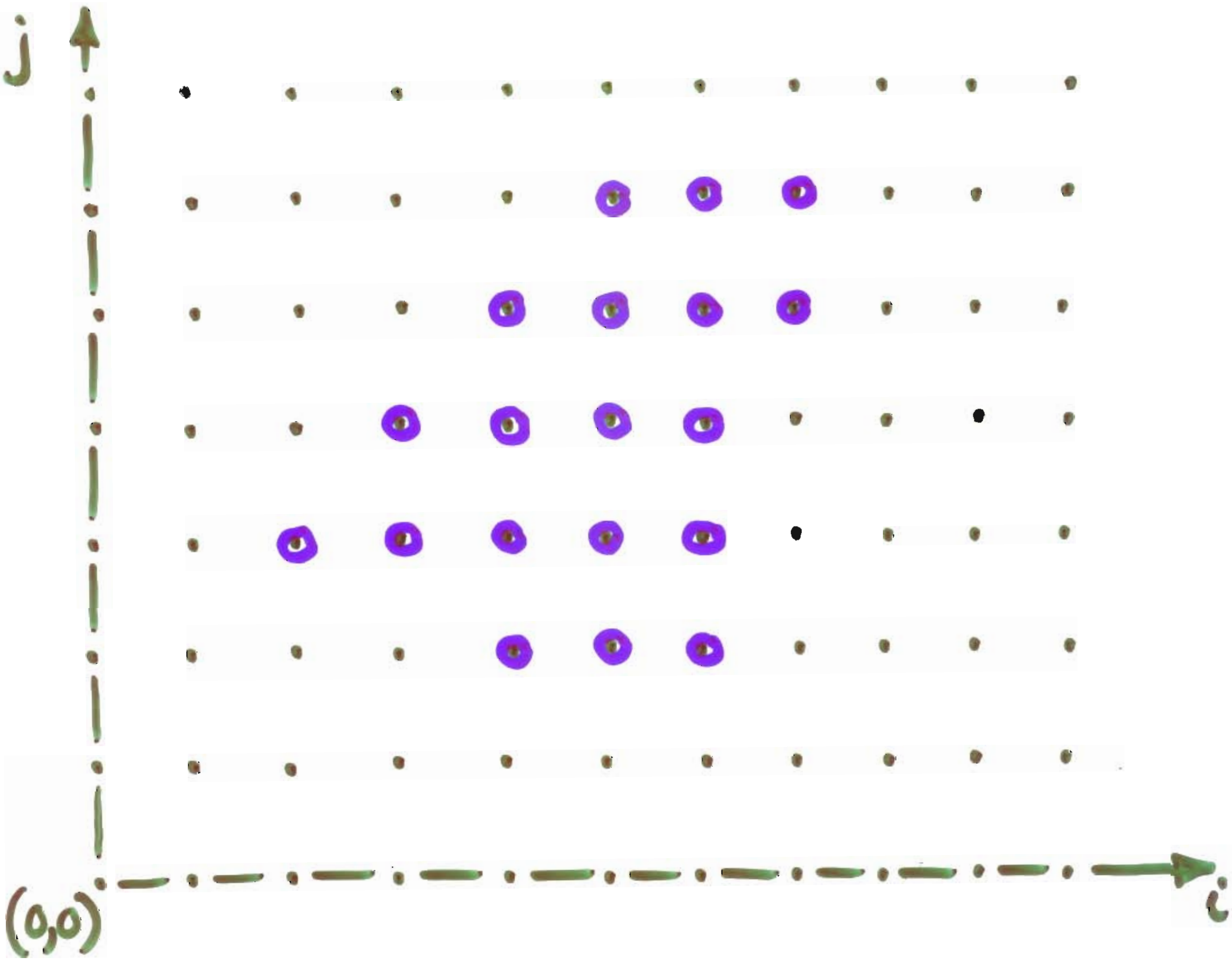
An analysis on how we estimate (X,Y) and how is the precision related to the shape S should also indicate

- HOW TO DESIGN SHAPES FOR HIGH-PRECISION LOCATION
- WHAT IS THE BEST POSSIBLE ACCURACY ACHIEVABLE

WE SHALL TRY TO ADDRESS THESE QUESTIONS DURING THIS LECTURE.

The shape S described by $\chi_S(x,y)$ in "master" coordinates





The "discrete" image of $S(X,Y)$

$$B(i,j) = \begin{cases} 1 & (\odot) \text{ if } (i,j) \in S(X,Y) \\ 0 & (\cdot) \text{ if } (i,j) \notin S(X,Y) \end{cases}$$

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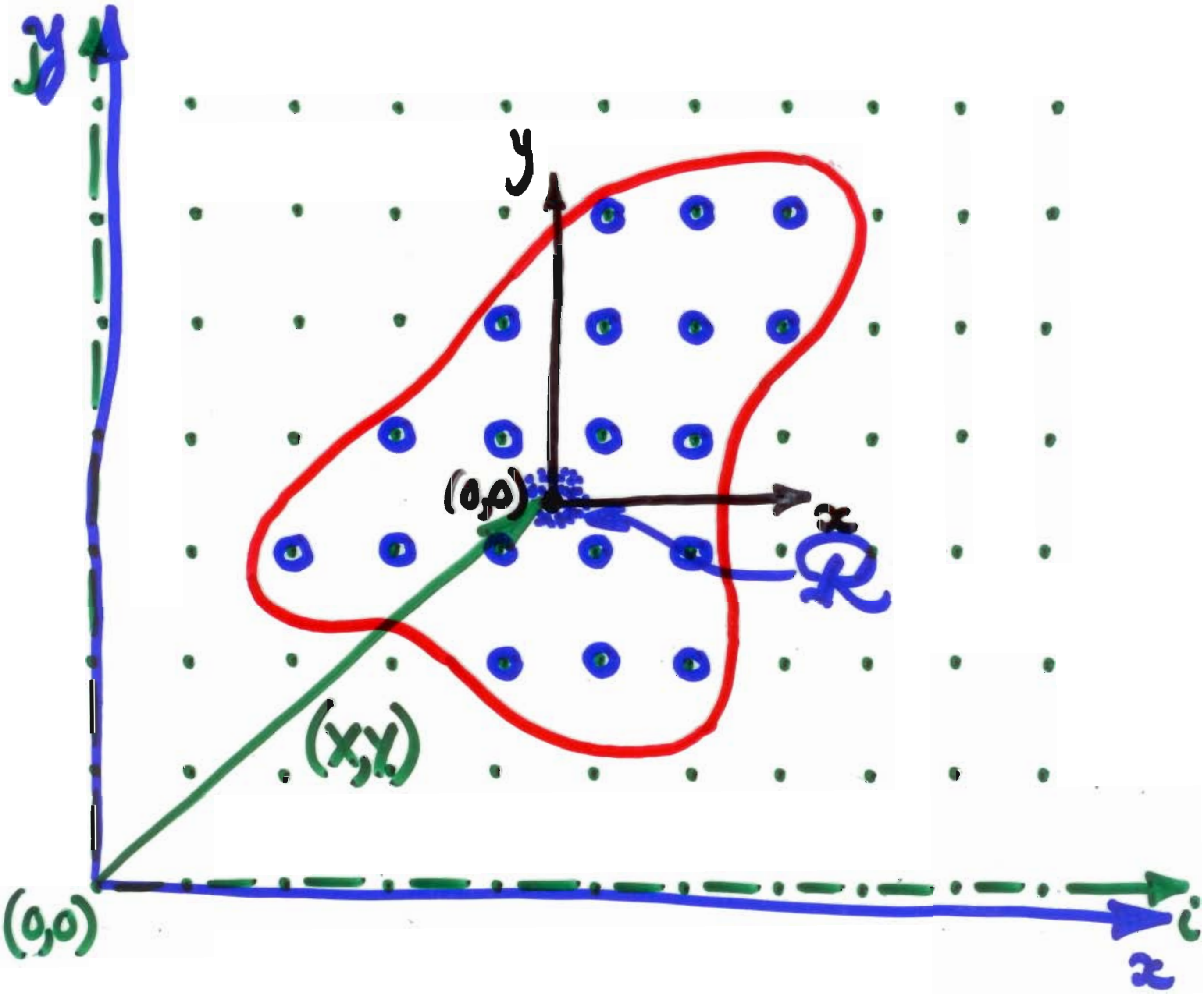

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ESTIMATING THE TRANSLATION

We have $\forall (i,j) \in \mathbb{Z}^2$

$$B(i,j) = \chi_S(i-X, j-Y) = \begin{cases} 1 & \text{or} \\ 0 \end{cases}$$

We assume full knowledge of the shape S (in its "master" coordinates), hence each value $B(i,j)$, $(i,j) \in \mathbb{Z}^2$, contrains the possible translation vectors.

Let $\mathcal{R}^B(i_0,j_0)$ be the region in $\mathbb{R}^2$ for which if $(X,Y) \in \mathcal{R}^B(i_0,j_0)$ we have the observed $B(i_0,j_0)$.

Then the (X,Y) for the given image $B(i,j)$ can belong to:

$$\bigcap \mathcal{R}^B(i_0,j_0) = \mathcal{R}^B$$

All (X,Y)-pairs that belong to $\mathcal{R}^B$ are EQUIVALENT from the CAMERA'S point of view (they are indistinguishable since they yield the same digitized IMAGE $B(i,j)$).

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Hence we can define the LOCATION PRECISION for S as the AREA of the region $\mathcal{R}^B$ in the worst case.

Given S find $\mathcal{R}^{WORST}_{(S)}$ and define

$$\text{Location Precision} = \text{Area}\{\mathcal{R}^{WORST}_{(S)}\}$$

(Note that $\mathcal{R}$ could look like 

 etc so some other measures can be contemplated!)

(and SHOULD!)

AN "INFORMATION THEORETIC" BOUND FOR THE LOCATION ACCURACY

- Suppose S has a finite support of $[0,A) \times [0,A)$ for some $A \in N^+$
- Suppose also that S is chosen so as to make it easy to determine (X,Y) to within a pixel, i.e., by "looking" at $B(i,j)$ we can determine that

$$(X,Y) \in (i_0-1, i_0] \times (j_0-1, j_0]$$

Then in $B(i,j)$ WE CAN HAVE AT MOST A^2 MEANINGFUL BITS!

[In fact $S(X,Y)$ and $S(X+m, Y+n)$ when digitized yield the same pattern up to an integer translation ($m,n \in N$ or $\mathbb{Z}$).]

But A^2 bits will code at most 2^{A^2} regions to which (X,Y) can belong hence the area of each such region must be BIGGER THAN

$$\frac{\text{PIXEL AREA}}{2^{A^2}} = \frac{1}{2^{A^2}} \leq \text{Area}\{R^{\text{WORST}}\}$$

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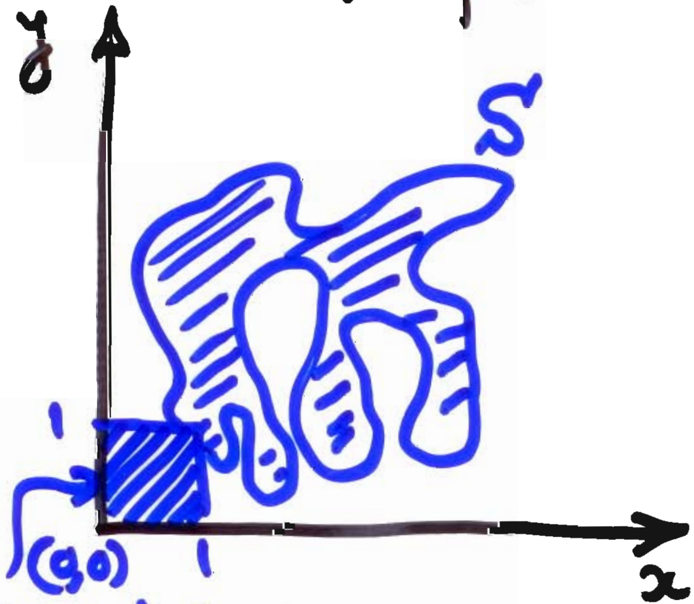
(otherwise all areas will not add up to the initial one pixel uncertainty area).

To ensure that we know (X,Y) to within a pixel we can allocate one bit to this, as follows



ROUGH LOCATION MARK

THEN $\quad \text{Area}\{\mathcal{R}^{\text{WORST}}\} \geq \dfrac{1}{2^{A^2-1}}$

A "balanced" design for S' should give us

$$\Delta X = \Delta Y \sim \sqrt{\frac{1}{2^{A^2-1}}}$$

for the precision of estimating (X,Y).

Given S' we can now evaluate it: it should not be too far from the best performance possible, i.e. should yield as small an R^{worst} as possible for its AREA.

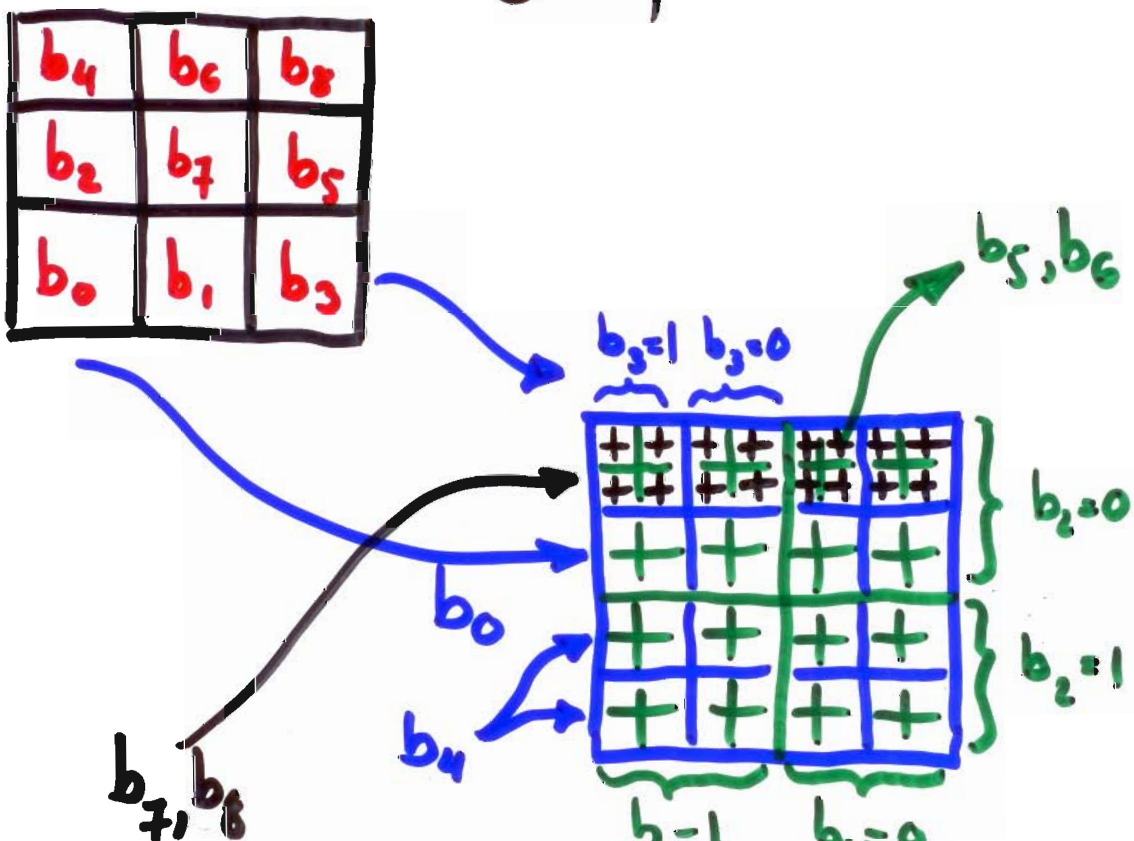
QUESTION: IS THERE AN OPTIMAL S'?

ANSWER: YES ..... BUT

The "OPTIMAL" LOCATION PATTERNS

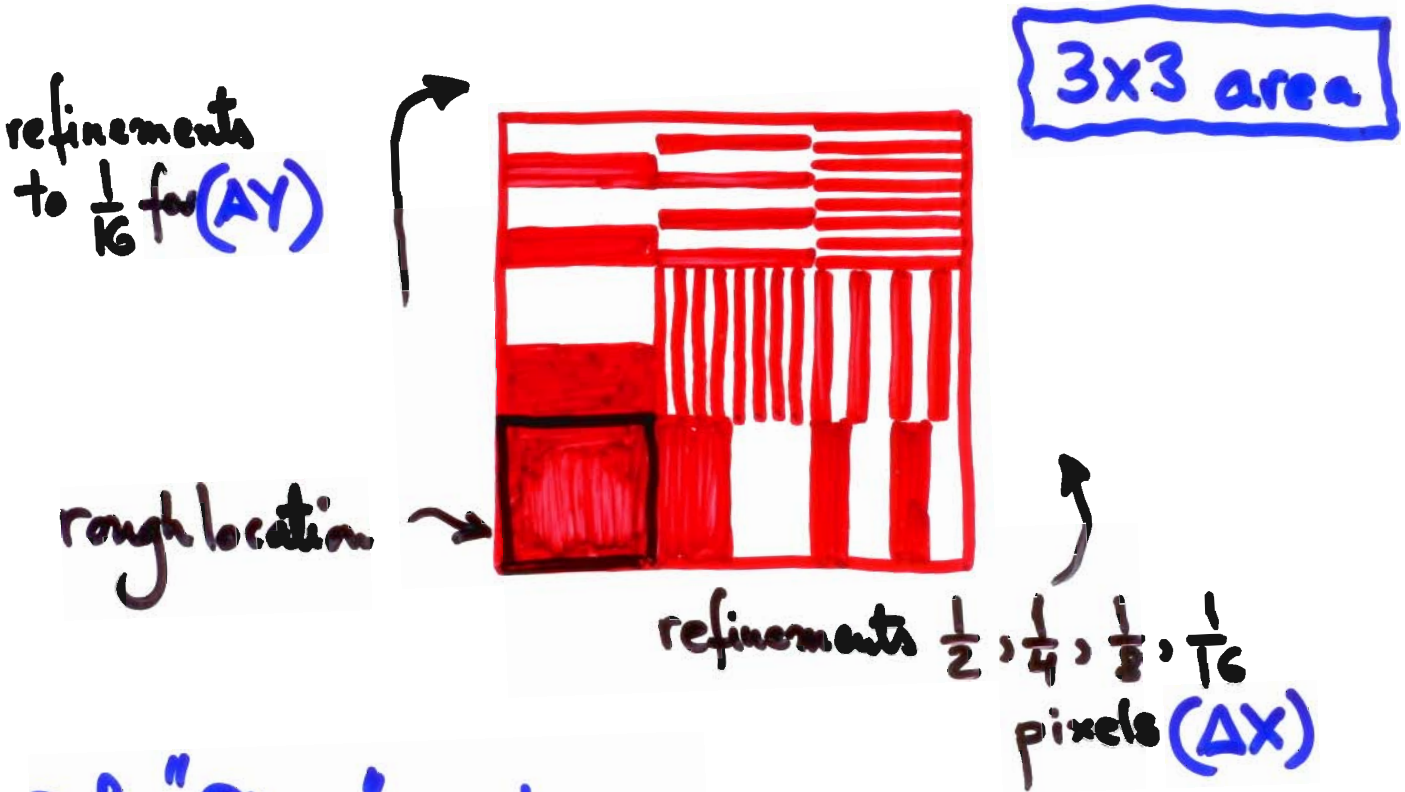
- has a rough location bit (lower left) + (A^2-1) information bits (8 in the example)

- for every bit-pattern the area of equivalent locations (ambiguity region $\mathcal{R}^B$) results from a recursive region partition:



(A QUAD-TREE DIVISION)

IT LOOKS LIKE THIS:



This "Shape" achieves

$$\Delta X = \Delta Y = \frac{1}{2^4} \text{ precision i.e.}$$

$$\text{Area}\{R^{\text{worst}}\} = \frac{1}{2^8} = \frac{1}{2^{3^2-1}}$$

This SHAPE requires precise knowledge of grid size and a TRANSLATION-only situation!

"Topology Preservation"

- If fiducial is

image should be

"similar".

i.e. same* connected components

same type of shape!!!

If we assume translation only & complete knowledge of the grid size we get precision of

$$\Delta X = \Delta Y \cong 1/A \quad \text{in area } 3A$$

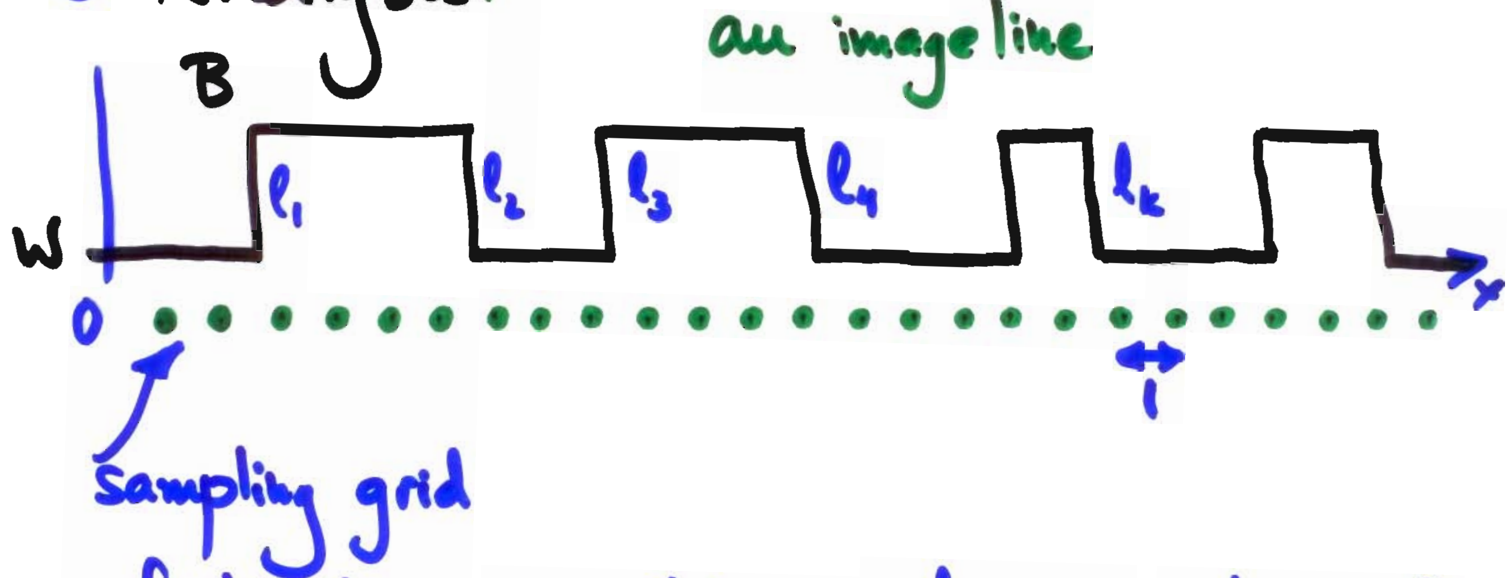
$$\left(\tfrac{1}{16} \quad \text{-------} \quad 48\right)$$

optimal

$$\Delta X = \Delta Y = \sqrt{\frac{1}{2^{A^2-1}}} \quad \text{in area } A^2$$

$$\left(\tfrac{1}{16} \quad \text{------} \quad 9\right)$$

- **Analysis:**



- for topology preservation: need runs > 1 (pixel)
- as we translate the shape jumps occur at

$$\varepsilon_k = 1 + \lfloor l_k \rfloor - l_k \quad \text{points}$$

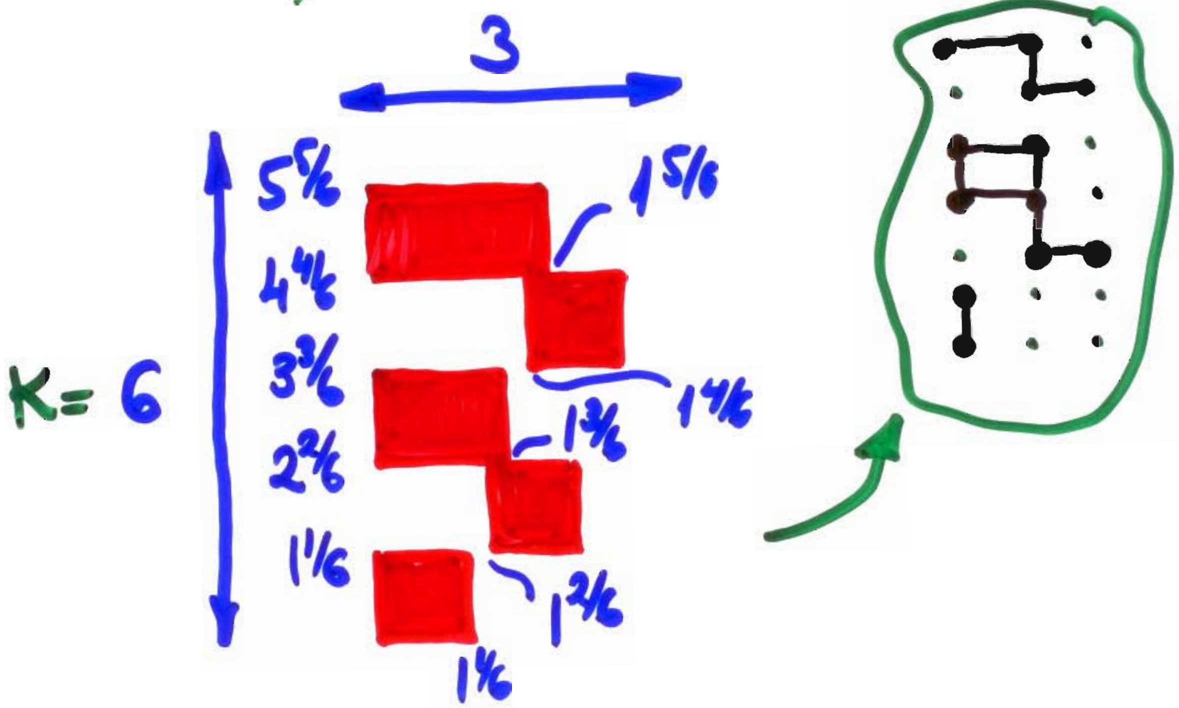
The various configurations of the type

00 1111 000111 0000 11 00 111 000

encode the intervals defined by the ε_k's.

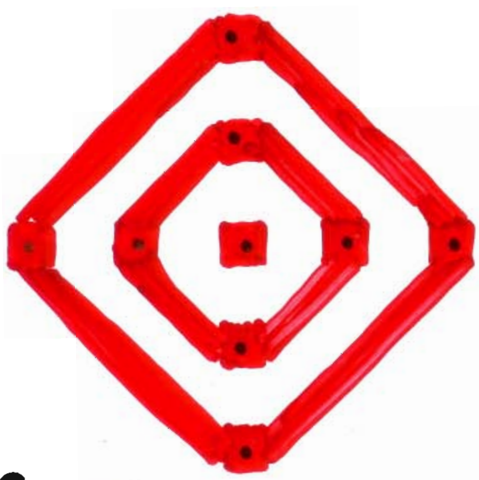
2D Analysis for patterns with 1D top. pres. intercepts.

A TOPOLOGY PRESERVING FIDUCIAL



area 18, precision $\Delta X = \Delta Y = \frac{1}{6}$.

Other possibilities



NOT GOOD (sloped lines have poor intercepts!)

MUCH BETTER

If we want rotational invariance with precise scale info (Euclidean case) we need

CIRCULAR SHAPES:

DISKS

BULLEYES

From the analysis so far we must conclude that BULLEYES, yielding richer digitized patterns are quite GOOD FIDUCIALS.

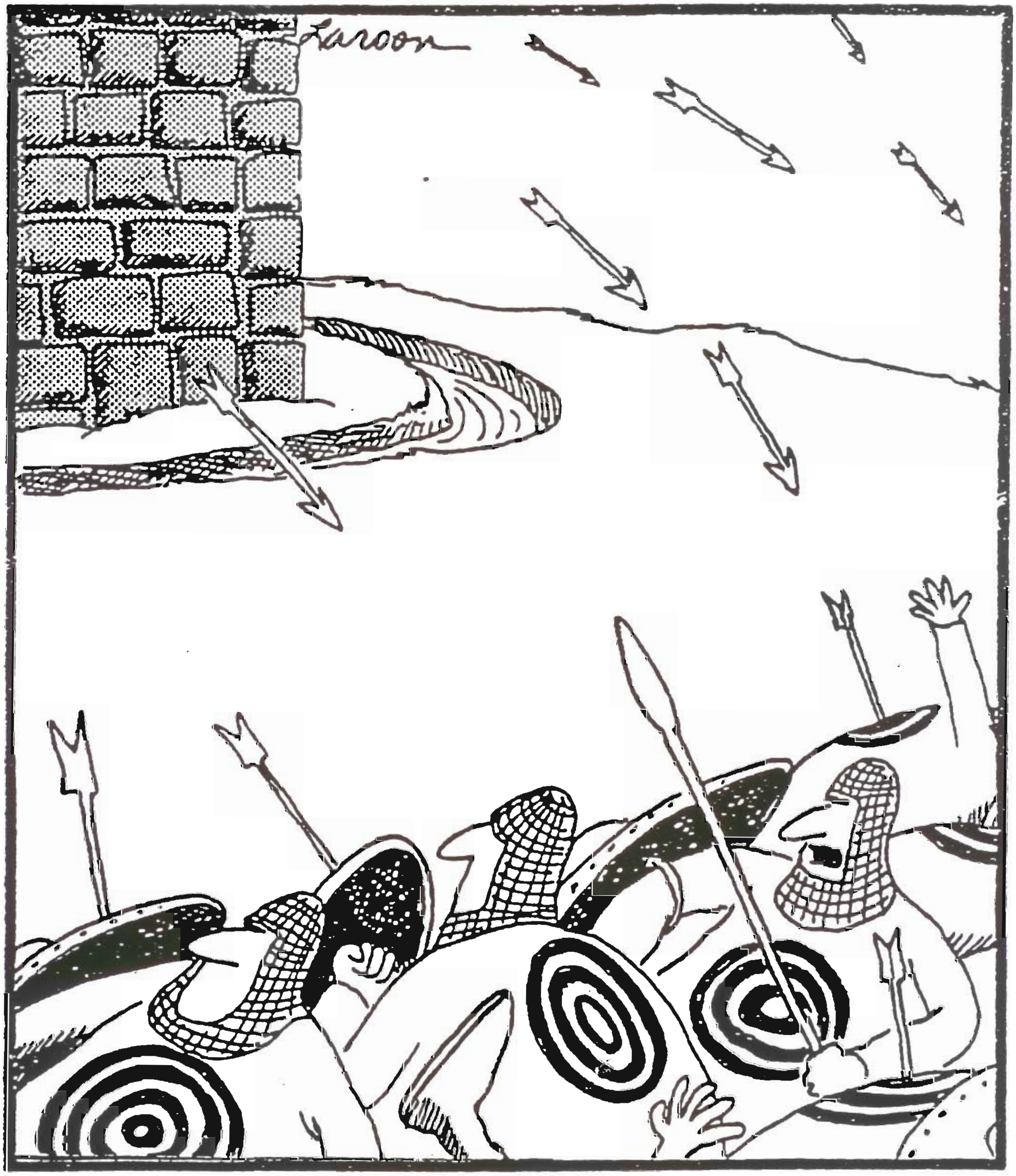
$\exists$ in the literature analyses of digitized disks from which the theoretical precision achieved with such fiducials can be computed.

This shows that the Royal Air Force chose rather poorly a deadly targeting shape to be painted on its planes during World War II. (See Figure 13.)

R.J. Mitchell's Supermarine Spitfire which first flew in 1936

Figure 13

A GOOD LOCATION MARK!



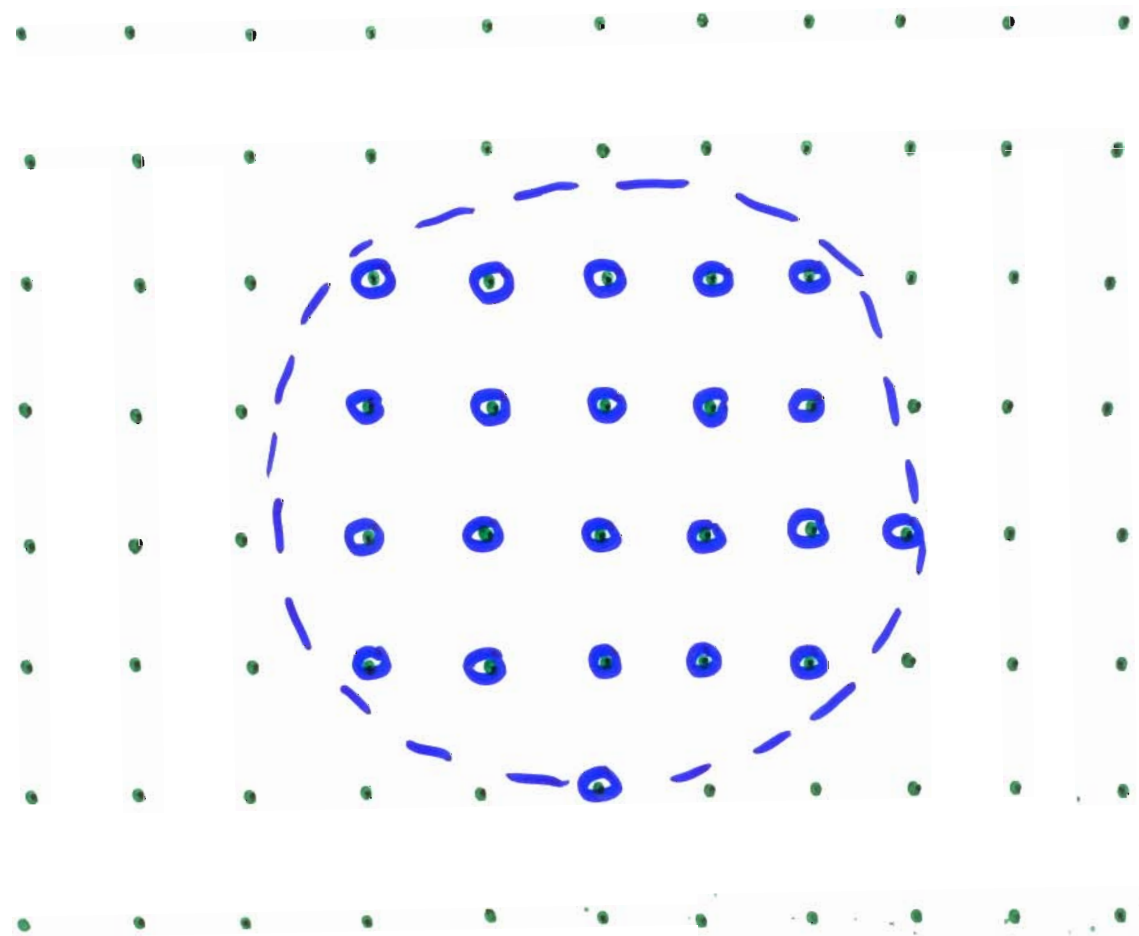
"What did I say, Boris? . . . These new uniforms are a crack!"

LARSON's opinion on it!

SIMILARITY INVARIANCE

→ Circular shapes with no RADIUS INFO.

Suppose we know that a DISK was digitized but we do not know the radius



Where can the centers of all possible "preimage" circles be?

If P is a possible center then

IT IS CLOSER TO ALL THE BLACK POINTS
THAN TO ANY WHITE POINT!

Hence in the COMPUTATIONAL GEOMETRY lingo
$P \in \mathcal{R}^B$ which is the cell corresponding to
B in the order-$\boxed{\mathrm{card}(B) = \#\text{black points}}$
VORONOI DIAGRAM of the UNIT GRID $\mathbb{Z}^2$.

$\mathcal{R}^B$ so defined is a convex region in $\mathbb{R}^2$.

$$\mathrm{Area}(\mathcal{R}^B) \sim \frac{1}{\mathrm{card}(B)} \quad \text{(LB)}$$

$\mathrm{card}\, B \sim r^2$ (r - unknown radius)

hence $\mathrm{Area}(\mathcal{R}^B) \sim \frac{1}{r^2}$ (LB)

Extensions:

for practical applications

- IMPROVE CAMERA MODEL
  - pixel area sampling (not point!)
  - camera provides GREY-LEVELS
- COMPUTATIONAL ASPECTS of REGISTRATION
  - compare with correlation methods
  - analyze approximate localization algorithms.
- "multiple" registration problems vs location (as discussed)
- affine distortions & SPACE FIDUCIALS. (in work now!)

Presentation based on:

- DESIGN of SHAPES for PRECISE REGISTRATION
  A.M. Bruckstein, L. O'Gorman, A. Orlitsky, AT&T TM, Oct 1989

- See also

  THE TOPOLOGY of LOCALES & POS. UNCERTAINTY
  D.I. Havelock, PAMI, Sept 1991, vol 13/4.

  SUBPIXEL REGISTRATION USING CIRCULAR FIDUCIALS
  A. Efrat & C Gotsman, IJCG&A, vol 4, 1994