IMPROVING THE VISION OF MAGIC EYES

A GUIDE TO BETTER AUTOSTEREOSCOPIC IMAGES

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AUTOSTEREOSGRAMS

single images that convey depth information

- invented by C.W. Tyler (in the seventies) following work on Random Dot Stereograms by Julesz (sixties)
- made very very popular recently by the "MAGIC EYE" book/poster/postcard craze

- some see DEPTH and ENJOY, others are convinced it is a world-wide CONSPIRACY!
- not all autostereograms are equally easy to "lock into", i.e. to interpret even for the trained ones
STEREOGRAMS - Image Pairs as seen by our Left & Right Eyes

$I_L(x) = \text{Surface Color at } x = A(x)$

$I_R(\bar{x}) = \text{Surface Color at } \bar{x} = A(Y(x))$

$I_R(x + \text{const}(P(x))) = I_L(x) = A(x)$
STEREOGRAMS - Image Pairs as seen by our Left and Right Eyes

Projection direction

\[ I_L(x) = \text{Surface Color at } x = A(x) \]
\[ I_R(\bar{x}) = \text{Surface Color at } \bar{x} = A(\psi(x)) \]
\[ I_R(x + \text{const} \phi(x)) = I_L(x) = A(x) \]
\[ \Delta = \text{const} \phi(x) - \text{DISPARITY} \]
**Auto Stereograms**

Surface Color chosen to have **IDENTICAL** Left & Right Images!

\[ I(x) = I(x + c(p(x))) = I(x) = A(x) \]

for all \( x \)

I over any interval \([x, p(x)]\) determines \( I() \) completely

(assuming, as we do, that \( p(x) \) is not too large)
AUTO STEREOGRAMS

Surface Color chosen to have IDENTICAL Left & Right Images!

\[ I(x + \text{const}(p(x))) = I(x) = A(x) \]

a functional equation involving the depth \( p(x) \)

(a) for all \( x \)

(b) for any interval \([x, p(x)]\) determines \( I(x)\) completely

(assuming, as we do, that \( p(x) \) is not too large)

for every horizontal line in \( I(x, y)\)
Conclusion: An autostereogram is fully specified by:

- a depth profile $\sigma(x,y)$
- the assumed viewing projections (we took parallel in directions R, L vertical)
- a basic color pattern $I(x,y)$ $x \in [0, \sigma(x)]$, $ty$

Hence there are many autostereograms for a given depth profile $\sigma(x)$. This leads to the question: How should we choose the basic pattern in order to see better the depth?

This is a signal design problem for an unspecified receiver/observer/interpreter.
The "receiver" is the human visual system. Lots of speculations/models a experimental evidence about its operation. A quick summary:

The visual system tries to do local matches, correlations of grey levels, e.g., gradients, zero crossings of Laplacians, etc. to evaluate disparities from which depth can be recovered, etc., etc. . . . (⇒ we are in TROUBLE!)

We want to design an image that best "fools" the visual system into a depth interpretation that is unnatural (the flat interpretation is the natural one!) without knowing too much about its depth interpretation processes.
Let us escape into generalities:

Concentrate on the 1D problem and assume that the brain computes from $I_x \ast I_z$ (in our case identical = I) a matching function $\Lambda(x, \tilde{x})$

$\Lambda(x, \tilde{x})$ - how well does $I(\tilde{x})$ at its neighborhood match $I(x)$ at its neighborhood

(on a scale from 0 to 1-perfect match)

\[ x \rightarrow I(\tilde{x}) \]

\[ \tilde{x} = x + (\rho(x) + \rho(x + c(x))) \]

\[ \tilde{x} = x + c \rho(x) \]

X -+ X

PLANAR INTERPRETATION

\[ \Lambda(x, \tilde{x}) = \Lambda(\tilde{x}, x) \text{ (must be!)} \]

\[ \Lambda(x, x) = 1 \]
Let us consider several possible $\Lambda$'s and analyze the structure of $\Lambda(x, \bar{x})$, in terms of the valleys and ridges of this function. Clearly we want to design $\Lambda$'s so as to make the "correct $\Lambda$" have ridges at $\bar{x} = x + c\phi(x)$ etc. and only there and to have valleys between the ridges that enable the interpretation mechanism to easily move between them.

- I leads to sharp ridges - difficult to leave the planar interpretation
- I leads to sharp ridges but valleys are surmountable
- I leads to blurred ridges no sharp depth perception
1. \( \Lambda(x,x) = \delta(I(x)-I(x)) = \begin{cases} 1 & \text{if equal} \\ 0 & \text{if } I(x) \neq I(x) \end{cases} \)

**Grey Level Equality Indicator**

Clearly for any basic pattern that is one to one over \([0, c\theta(0)]\) we have an ideal \( \Delta(x,x) \) with ridges at the correct locations a zero everywhere else.

**Let us see such an image:**

![Figure 3a, \( I(x) \)](image)

and two more obtained by cutting this basic pattern into equal pieces a randomly permuting them

- Figure 3b, 3c (coarse afine)

From the assumed \( \Delta \)'s point of view all three are **equivalent** - so this \( \Delta \) is no good!
2. $\Lambda(x, x') = f\left( [I(x) - I(x')]^2 \right)$

$f(z); f(0) = 1, \text{monotone decreasing with } z \text{ towards 0}$

Examples $f(z) = 1/(1 + \lambda z), f(z) = e^{-\lambda z}$ etc...

Squared error function / grey level based!

Here we have ridges of max values (1) at the correct disparities.

$\Lambda(x, x')$

The sharpness of ridges determined by the Laplacian.

and

$$\nabla^2 \Lambda(x, x') \bigg|_{z = x + c p(x)} = 2f(0) \frac{1}{(\frac{d}{dx} I(x))^2 + (\frac{d}{dy} I(x))^2}$$

So for sharp edges we want high derivatives in $I(x)$, but we also want few accidental matches that will occur, if we have finite range for $I$. 
This leads toward the consideration of using random patterns assuming a $\Lambda(x, z)$ based on averaging.

Indeed we have many lines in the image that will encode nearly the same depth profile.

3. $\Lambda(x, z) = \int (E_{\omega}(I(x)) - I(x))^2$

$I(x)$ over $[0, p(0)]$, a realization of $I_{\omega}(x)$

Here "false match"-peaks are averaged out and disappear as the true ridges have sharpness given by

$\nabla^2 \Lambda(x, z) = 2f(0) R(0) \left[ \frac{A(x, B(x))}{z} \right]^2 + \left[ \frac{A(x, B(x))}{z} \right]^2$

$R(0)$ the autocorrelation of the $I_{\omega}$ process

(UNDER OUR CONTROL!)

all other factors are NOT under our control.
EXAMPLE:

\[ I_\omega(x) - \text{periodic process} = \sum_{i=0}^{\infty} a_i \cos(i\omega_0 x + \phi_i) \]

\[ R(\tau) = \frac{1}{2} \sum_{i=0}^{\infty} E(a_i^2) \cos(i\omega_0 \tau) \]

\[ R''(\tau) = -\frac{1}{2} \sum_{i=0}^{\infty} E(a_i^2)(i\omega_0)^2 \cos(i\omega_0 \tau) \]

If we filter out all frequencies beyond \( \sigma \omega_0 = K \omega_0 \) (LPF) we have \( (\sigma = \frac{1}{\zeta}) \)

\[ R''_{\sigma}(\tau) = -\frac{1}{2} \sum_{i=0}^{\infty} E(a_i^2)(i\omega_0)^2 \cos(i\omega_0 \tau) \]

as \( \sigma > 0 \) \( (K \gg \omega_0) \) the behavior of \( R''(\sigma) \)

is controlled by

\[ R''_{\sigma}(0) = -\frac{1}{2} \sum_{i=0}^{\infty} E(a_i^2)(i\omega_0)^2 \]
If $Ea_i^2 = \text{const}$ ("white noise" case)

$$R''_0(0) = -\frac{1}{2} \sum (\text{const} \, \omega_i^2) \, i^2 \propto \sigma^{-3}$$

If $Ea_i^2 = \frac{1}{2} i$ ("1/f noise" case)

$$R''_0(0) = -\frac{1}{2} \sum (\text{const} \, \omega_i^2) \, i \propto \sigma^{-2}$$

If $Ea_i^2 = \frac{1}{2} i^2$ ("1/f^2 noise" case)

$$R''_0(0) = -\frac{1}{2} \sum (\text{const} \, \omega_i^2) \propto \sigma^{-1}$$

If $Ea_i^2 = \frac{1}{2} i^3$ ("1/f^3 noise" case)

$$R''_0(0) = -\frac{1}{2} \sum (\text{const} \, \omega_i^2) \, i^{-1} \propto \ln(\sigma^{-1})$$

If $Ea_i^2 = \frac{1}{2} i^{3+\varepsilon}$ ("1/f^{3+\varepsilon} noise" case)

$$R''_0(0) = -\frac{1}{2} \sum (\text{const} \, \omega_i^2) \, i^{-(3+\varepsilon)} \propto \text{constant}$$

If we would measure distances with a particle proportional to the scale, i.e. $\tau \sigma \rightarrow \tau$ we would have

$$\frac{d^2 \, R_0(\tau)}{d(\tau \sigma)^2}, \text{i.e. } \sigma^2 R''_0(\tau)$$

For 1/f noise we'll have constant $\sigma^2 R''_0(0)$ at all scales. **BASIN of ATTRACTION scales with RESOLUTION** or **SCALE SPACE SELF SIMILARITY!!**