ON FIDUCIALS

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Fiducial comes from

fido (Latin) - to trust
    to rely upon

A fiducial mark
is an object that
enables self location
or registration
when "looked at" via a
sensing device

Imaging
Natural Fiducials

"old style GPS"

Celestial Navigation
Landmarks in the landscape are location navigation fiducials

(in conjunction with Maps - the prior info!)
Man-made fiducials

objects designed to
make self-location
and navigation and
registration as easy
as possible.

- Targeting Shapes
- Location Patterns
- Sign Posts
Sign Posts

* "You are here" signs on Street Maps

* Road Signs with distances to points of interest/destinations
You Are Here!

http://go.to/funpic
INFORMATION

YOU ARE HERE
Targeting Marks used to find, locate objects of interest!

Many applications for:
- Object Detection
- Identification (Bar Codes)
- Tracking
- Alignment
Bull's Eye: "the Ultimate Mark"

How good is it? Why is it good?
A Larson cartoon:

"Bummer of a Birthmark, Hal"
A quite useless fiducial!
Fiducial marker

A fiducial marker or fiducial is an object placed in the field of view of an imaging system which appears in the image produced, for use as a point of reference or a measure. It may be either something placed into or on the imaging subject, or a mark or set of marks in the reticle of an optical instrument.

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Accuracy

Applications
- Physics
- Geographical survey
- Augmented reality
- Metrology
- Fiducial marker sets
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  - Radio therapy
- Printed circuit boards
- Printing

See also

References

Accuracy

In high-resolution optical microscopy, fiducials can be used to actively stabilize the field of view. Stabilization to better than 0.1 nm is achievable.[1]

Applications

Physics

In physics, 3D computer graphics, and photography, fiducials are reference points: fixed points or lines within a scene to which other objects can be related or against which objects can be measured. Cameras outfitted with Réseau plates produce these reference marks (also called Réseau crosses) and are commonly used by NASA. Such marks are closely related to the timing marks used in optical mark recognition.

Geographical survey

Airborne geophysical surveys also use the term “fiducial” as a sequential reference number in the measurement of various geophysical instruments during a survey flight. This application of the term evolved from air photo frame numbers that were originally used to locate geophysical survey lines in the early days of airborne geophysical surveying. This method of positioning has since been replaced by GPS, but the term “fiducial” continues to be used as the time reference for data measured during flights.

Augmented reality

In applications of augmented reality, fiducials help resolve several problems of integration between the real world view and the synthetic images that augment it. Fiducials of known pattern and size can serve as real world anchors of location, orientation and scale. They can establish the identity of the scene or objects within the scene. For example, a fiducial printed on one page of an alternative reality popup book would identify the page to allow the system to select the augmentation content. It would also serve to moor the coordinates of the augmented content to the three dimensional location, orientation and scale of the open book, helping to create a stable and accurate fusion of real and synthetic imagery.

A slightly more complex example would be multiple fiducials, each attached to an individual piece in an alternative reality board game.

Metrology

The appearance of markers in images may act as a reference for image scaling, or may allow the image and physical object, or multiple independent images, to be correlated. By placing fiducial markers at known locations in a subject, the relative scale in the produced image may be determined by comparison of the locations of the markers in the image and subject. In applications such as photogrammetry, the fiducial marks of a surveying camera may be set so that they define the principal point, in a process called “collimation”. This would be a creative use of how the term collimation is conventionally understood.
Fiducial marker sets

Some barcode readers can estimate the translation, orientation, and vertical depth of a known-size barcode relative to the barcode reader.[2]

Some sets of fiducial markers are specifically designed to allow rapid, low-latency detection of the 2D location, 2D orientation, and identity of hundreds of unique fiducial markers. For example, the " amoeba" reactTVision fiducials, the d-touch fiducials,[3][4] or the TRIP circular barcode tags (ringcodes).[5]

Medical imaging

Fiducial markers are used in a wide range of medical imaging applications. Images of the same subject produced with two different imaging systems may be correlated by placing a fiducial marker in the area imaged by both systems. In this case, a marker which is visible in the images produced by both imaging modalities must be used. By this method, functional information from SPECT or positron emission tomography can be related to anatomical information provided by magnetic resonance imaging (MRI).[6] Similarly, fiducial points established during MRI can be correlated with brain images generated by magnetoencephalography to localize the source of brain activity. Such fiducial points or markers are often created in tomographic images such as computed tomography, magnetic resonance and positron emission tomography images using a device known as the N-localizer.[8][9][10][11][12][13][14][15][16][17][18]

Electrocardiography

In electrocardiography (ECG), fiducial points are landmarks on the ECG complex such as the isoelectric line (PQ junction), and onset of individual waves such as PQRST.

Cell biology

In processes that involve following a labelled molecule as it is incorporated in some larger polymer, such markers can be used to follow the dynamics of growth/shrinkage of the polymer, as well as its movement. Commonly used fiducial markers are fluorescently labelled monomers of bio-polymer. The task of measuring and quantifying what happens to these is borrowed from methods in physics and computational imaging like Speckle imaging.

Radiotherapy

In radiotherapy and radiosurgical systems, fiducial points are landmarks in the tumour to facilitate correct targets for treatment. In neuro-navigation, a "fiducial spatial coordinate system" is used as a reference, for use in neurosurgery, to describe the position of specific structures within the head or elsewhere in the body. Such fiducial points or landmarks are often created in magnetic resonance imaging and computed tomography images by using the N-localizer.

Printed circuit boards

In printed circuit board (PCB) manufacturing, fiducial marks, also known as circuit pattern recognition marks, allow SMT placement equipment to accurately locate and place parts on boards. These devices locate the circuit pattern by providing common measurable points. They are usually made by leaving a circular area of the board bare from solder-mask coating. Inside this area is a circle exposing the copper plating beneath. This center metallic disc can be solder-coated, gold-plated or otherwise treated, although bare copper is most common if not a current-carrying contact.

Most placement machines are fed boards for assembly by a rail conveyor, with the board being clamped down in the assembly area of the machine. Each board will clamp slightly differently than the others, and the variance—which will generally be only tens of a millimeter—is sufficient to ruin a board without proper calibration. Consequently, a typical PCB will have multiple fiducials to allow placement robots to precisely determine the board’s orientation. By measuring the location of the fiducials relative to the board plan stored in the machine’s memory, the machine can reliably compute the degree to which parts must be moved relative to the plan, called offset, to ensure accurate placement.

Using three fiducials enables the machine to determine offset in both the X and Y axes, as well as to determine if the board has rotated during clamping, allowing the machine to rotate parts to be placed to match. Parts requiring a very high degree of placement precision, such as ball grid array packages, may have additional fiducials near the package placement area of the board to further fine-tune the targeting.

Conversely, low end, low-precision boards may only have two fiducials, or use fiducials applied as part of the screen printing process applied to most circuit boards. Some very low-end boards may use the plated mounting screw holes as ersatz fiducials, although this yields very low accuracy.

For prototyping and small batch production runs, the use of a fiducial camera can greatly improve the process of board fabrication. By automatically locating fiducial markers, the camera automates board alignment. This helps with front to back and multilayer applications, eliminating the need for set pins.[10]

Printing

In color printing, fiducials—also called “registration black”—are used at the edge of the cyan, magenta, yellow and black (CMYK) printing plates so that they can be correctly aligned with each other.

See also

- Secchi disk
- Landmark point

https://en.wikipedia.org/wiki/Fiducial_marker
Printing Registration Marks

Multiple layers of printing on a surface must be correctly aligned.

Applications in
- Color printing
- VLSI production
- P.C.B.'s for Electronics
Before printing of an order is started the printer needs to be calibrated. A normal commercial printer can have several ink containers (2 colors, 4 colors, 5 colors, 6 colors etc.) and a printing job needs a basic of 4 colors of CMYK.

Since these four colors are printed at four different printing heads (rolls) the output print may not be perfectly aligned. And if the four heads are not perfectly calibrated, the final print out will not have the perfect colored image as required.

To calibrate the printing heads a sample set is printed. Each page/sheet/paper printed will have a small 'target' mark (as shown above) on four sides of the paper.
from a layer alignment/registration patent!

FIG. 2

ALIGNMENT WINDOW
ALIGNMENT CAVITY

FIG. 3

FIG. 4

TOP ALIGNMENT WINDOW
BOTTOM ALIGNMENT MARKS
ALIGNMENT CAVITY
A THEORY for FIDUCIAL DESIGN

- A model for the fiducial object
  - Example: 1) A planar shape
  - 2) A constellation of points
  - 3) A 3D object with colored surface

- A model for the imaging process
  - Example: 1) The digitization process induced by pixel sampling
  - 2) The 3D → 2D camera projection

- A precise definition of the desired information needed from the data
Example

1) the translation in the plane \((Ax, Ay)\)

2) the relative 3D orientation (wrt the fiducial)

The Design Goal:
Best precision of estimation (for the desired information) to be achieved by optimizing the fiducial object given the known imaging process.

We want optimal fiducials.
Example

- Design of planar shape fiducials
  for optimal location/translation estimation
  under point sampling digitization
DESIGN OF PLANAR SHAPES 
FOR PRECISE REGISTRATION

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AT&T-BELL LABORATORIES

joint work with A. Orlitsky & J. O'Gorman
Problem: Design fiducials that enable subpixel image registration

Applications

- Automatic visual inspection of printed circuit boards or VLSI silicon wafers
- Location problems in robotic assembly or soldering
- Overlay placement in etching, printing, etc... etc...
Fiducials are widely used. They come in many shapes:

They are viewed by cameras that sample and quantize the image so it makes sense to ask, given some knowledge of the camera operation, what are the shapes that yield best location accuracy.
The Mathematical Formalization:

- **Fiducial Image is Binary (B/W)**
  with shape described via

\[ X_S(x,y) = \begin{cases} 
  1 & (x,y) \in S, \text{ ie } I(x,y) = B \\
  0 & (x,y) \notin S, \text{ ie } I(x,y) = W 
\end{cases} \]

- **Translated version of S, by (X,Y)**, are
digitized by sampling on the Unit Grid
\( \{(i,j) \in \mathbb{Z}^2\} \), yielding binary images

\[ S(i,j) = \begin{cases} 
  1 & (i,j) \in S(x,y) \\
  0 & (i,j) \notin S(x,y) 
\end{cases} = X_S(x,y)(ij) = X_S(i-x,j-y) \]
Suppose we are given the shape $S$ (in its "master" coordinates) and the digitized image $B(i,j)$ of a translated version of $S$.

1) How should we estimate the translation $(x,y)$ from $B(i,j)$?

2) What is the precision in estimating $(x,y)$, what influences this precision?
An analysis on how we estimate \((X,Y)\) and how is the precision related to the shape \(S\) should also indicate

- **How to design shapes for high-precision location**

- **What is the best possible accuracy achievable**

We shall try to address these questions during this lecture.
The shape $S$ described by $x(G,y)$ in "master" coordinates.
The "discrete" image of $S(x,y)$

$B(i,j) = \begin{cases} 
1 & \text{if } (ij) \in S(x,y) \\
0 & \text{if } (ij) \notin S(x,y) 
\end{cases}$
The shape $S$ described by $x(f(x,y))$ in "master" coordinates.

The "discrete" image of $S(x,y)$

$$B(i,j) = \begin{cases} 1 & (i,j) \in S(x,y) \\ 0 & \text{(otherwise)} \end{cases}$$
**ESTIMATING THE TRANSLATION**

We have \( f(c_{ij}) \in \mathbb{Z}^2 \)

\[ B(c_{ij}) = X_s(i-x, j-y) = \{ 1 \text{ or } 0 \} \]

We assume full knowledge of the shape \( S \) (in its 'master' coordinates), hence each value \( B(c_{ij}) \), \((i,j) \in \mathbb{Z}^2\), constrains the possible translation vectors.

Let \( R(c_{i_0,j_0}) \) be the region in \( \mathbb{R}^2 \) for which if \((x,y) \in R(c_{i_0,j_0})\) we have the observed \( B(c_{i_0,j_0}) \).

Then the \((x,y)\) for the given image \( B(c_{ij}) \) can belong to:

\[ \bigcap R(c_{i,j}) = \mathbb{R}^3 \]

All \((x,y)\)-pairs that belong to \( \mathbb{R}^3 \) are EQUIVALENT from the CAMERA'S point of view (they are indistinguishable since they yield the same digitized IMAGE \( B(c_{ij}) \)).
All \((X,Y)\)-pairs that belong to \(R^3\) are EQUIVALENT from the CAMERA'S point of view (they are indistinguishable since they yield the same digitized IMAGE \(B(ij)\)).

Hence we can define the LOCATION PRECISION for \(S\) as the AREA of the region \(R^3\) in the worst case.

Given \(S\) find \(R_{\text{Worst}}(S)\) and define

\[
\text{Location Precision} = \text{Area}(R_{\text{Worst}}(S))
\]

(Note that \(R\) could look like \(\bullet\) etc so some other measures can be contemplated!)

(And SHOULD!)
An "Information Theoretic" Bound
For the Location Accuracy

- Suppose $S'$ has a finite support of $[0,A) \times [0,A)$ for some $A \in \mathbb{N}^+$
- Suppose also that $S'$ is chosen so as to make it easy to determine $(x,y)$ to within a pixel, i.e., by looking at $B(i,j)$ we can determine that $(x,y) \in (i-1,i] \times (j-1,j)$

Then in $B(i,j)$ we can have at most $A^2$

Meaningful Bits!

[In fact $S(x,y)$ and $S(x+m,y+n)$ when digitized yield the same pattern up to an integer translation $(m,n \in \mathbb{N} \cup \mathbb{Z})$.]

But $A^2$ bits will code at most $2^{A^2}$ regions to which $(x,y)$ can belong hence the area of each such region must be BIGGER THAN

$$\frac{\text{Pixel Area}}{2^{A^2}} = \frac{1}{2^{A^2}} \leq \text{Area}\text{for } x$$
But $A^2$ bits will code at most $2^{A^2}$ regions to which $(X,Y)$ can belong hence the area of each such region must be BIGGER THAN

$$\frac{\text{Pixel Area}}{2A^2} = \frac{1}{2A^2} \leq \text{Area of } R_{\text{worst}}$$

(otherwise all areas will not add up to the initial one pixel uncertainty area).

To ensure that we know $(X,Y)$ to within a pixel we can allocate one bit to this, as follows.

ROUGH LOCATION MARK

THEN $\text{Area of } R_{\text{worst}} \geq \frac{1}{2A^2}$
A "balanced" design for $S$ should give

$$\Delta X = \Delta Y \sim \sqrt{\frac{1}{2^{A^2-1}}}$$

for the precision of estimating $(X,Y)$.

Given $S$ we can now evaluate it: it should not be too far from the best performance possible, i.e. should yield as small an $R_{\text{work}}$ as possible for its area.

**Question:** Is there an optimal $S$?

**Answer:** Yes

**But**
The "Optimal" Location Patterns

- has a rough location bit (lower left) + \((A^2-1)\) information bits (8 in the example)

- for every bit-pattern the area of equivalent locations (ambiguity region \(R^b\)) results from a recursive region partition:

(A Quad-Tree Division)
This looks like this:

Refinements to $\frac{1}{16}$ for $(\Delta Y)$

Rough location

Refinements $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ pixels ($\Delta X$)

This shape achieves

$$\Delta X = \Delta Y = \frac{1}{2^n}$$

Precise, i.e.

$$\text{Area of } R \text{ work } j = \frac{1}{2^8} = \frac{1}{2^{3+1}}$$

This shape requires precise knowledge of grid size and a translation-only situation!
"Topology Preservation"

- If fiducial is

  \[ \text{image should be similar.} \]

  i.e. same connected components

  same type of shape!!!

If we assume translation only a complete knowledge of the grid size we get precision

\[
\Delta x = \Delta y = \frac{1}{A} \text{ in area } 3A
\]

optimal

\[
\Delta x = \Delta y = \sqrt{\frac{1}{2^A^2 - 1}} \text{ in area } A^2
\]

\[
\left(\frac{1}{16} \quad \cdots \quad 48\right)
\]

\[
\left(\frac{1}{16} \quad \cdots \quad 9\right)
\]
• Analysis:

- Sampling grid
- For topology preservation: need runs > 1 (pixel)
- As we translate the shape jumps occur at
  \[ E_k = 1 + \text{length of } l_k \] points

The various configurations of the type

001111 000111 000011 001111000

Encode the intervals defined by the \( E_k \)'s.

2D Analysis for patterns with 1D top. pres. intercepts.
A topology preserving fiducial

\[ \kappa = 6 \]

area 18, precision \( \Delta X = \Delta Y = \frac{1}{6} \).

Other possibilities

[Diagrams of different shapes]

Not good (slope lines have poor intercepts!)

MUCH BETTER
If we want rotational invariance with precise scale info (Euclidean case) we need **CIRCULAR SHAPES:**

- **Disks**
- **Bulleyes**

From the analysis so far we must conclude that **Bulleyes**, yielding richer digitized patterns are quite **GOOD FIDUCIALS.**

In the literature analyses of digitized disks from which the theoretical precision achieved with such fiducials can be computed.
This shows that the Royal Air Force chose rather poorly a deadly targeting shape to be painted on its planes during World War II. (See Figure 13.)

R.J. Mitchell's Supermarine Spitfire which first flew in 1936

Figure 13

A good location mark!
"What did I say, Boris? ... These new uniforms are a crock!"

~ Larson's opinion on it!
Similarity Invariance

Circular shapes with no radius information.

Suppose we know that a disk was digitized, but we do not know the radius.

Where can the centers of all possible "preimage" circles be?
If $P$ is a possible center then

it is closer to all the black points
than to any white point!

Hence in the computational geometry lingo
$P \in \mathbb{R}^3$ which is the cell corresponding to
$B$ in the order - $\text{card}(B) = \# \text{black points}$

Voronoi diagram of the unit grid $\mathbb{Z}^2$.

$\mathbb{R}^3$ so defined is a convex region in $\mathbb{R}^2$.

\[
\text{Area}(\mathbb{R}^3) \sim \frac{1}{\text{card}(B)} \quad (18)
\]

\[
\text{card } B \sim r^2 \quad (r - \text{unknown radius})
\]

hence \( \text{Area}(\mathbb{R}^3) \sim \frac{1}{r^2} \quad (18) \)
Extensions:
  for practical applications

- Improve camera model
  - Pixel area sampling (not point!)
  - Camera provides grey-levels

- Computational aspects of registration
  - Compare with correlation methods
  - Analyze approximate localization algorithms

- "Multiple" registration problems
  vs location (as discussed)

- Affine distortions vs space fiducials
  (in work now!)
Presentation based on:

- Design of SHAPES for Precise Registration  
  AM Bruckstein, LGerman, AO Nitzky, ATAT TM, Oct 1989

See also

- The Topology of Locales a Prob. Uncertainty  

- Subpixel Registration using Circular Fiducials  
  A. Efros and C. Gotsman, IJCGAA, vol 4, 1994
A PRACTICAL CONSEQUENCE

KLA-Tencor's AIM Targets
KLA Tencor announces Archer 200 overlay metrology system

by M. David Levenson, Editor-in-Chief, Micro lithography World

As the semiconductor industry advances toward the 32nm node, overlay specifications have narrowed dramatically, requiring control of high-order grid and field distortions, according to the International Technology Roadmap for Semiconductors (ITRS). Seeking to meet the new requirements is KLA-Tencor's new Archer 200, the latest version of the company's imaging overlay measurement tool.

Boasting a total machine uncertainty (TMU) of 1nm (25% better than the Archer 100) and 25% better throughput, Archer 200 will facilitate high order overlay control for all three exposure tool vendors, according to Noam Knoll, VP of marketing for the company's overlay group, who gave an overview of the new tool for WorkerNEWS. The Archer 200 uses the KLA-Tencor AIM grating alignment mark and is compatible with 10μm² μAIM targets as well.

Example of overlay error. (Source: KLA-Tencor)

Overlay performance good enough for double patterning requires accurate modeling of displacements and distortions, including higher order errors within the field. Accurate modeling requires taking more measurements — for example, Knoll estimated that 3rd order field distortion requires six targets within the exposure field to capture adequately. The use of μAIM targets minimizes the loss of useful real-estate. Capturing the higher-order grid and field distortions (with several hundred measurements/wafer) reduces the residual errors by 40%, according to Knoll. With a more-acquire-measurement (MAM) time of 0.65sec (20% faster), the Archer 200 maintains a throughput of 100 wafers/hr. In a statement, KLA-Tencor listed the redesigned systems' other improvements, including a 20% improved "tool induced shift" of 0.8nm and 30%-50% better matching, with unspecified "greater measurement repeatability."

http://electroiq.com/blog/2008/06/kla-tencor-announces-archer-200-overlay-metrology-system/
KLA-Tencor

AIM fiducials
GRAND CHALLENGE:

Use Deep Convolutional Neural Networks-based Learning to Design Optimal Fiducials
Conjecture 1: Let \((m,n)\) denote the linear span of \(S_m\), where \(S_m\) is the 0.1 sequence from the Segre hyperoval defined in the last paragraph. Then

\[
\ell(m,n) = 1 + (m - 2)/(m - 2) + (m - 4)/(m - 4)
\]

for all \(m \geq 0\).

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References


Design of Shapes for Precise Image Registration

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Abstract—This correspondence deals with the problem of designing planar shapes for subpixel image registration. Basic theoretical considerations are shown to lead to a lower bound on location accuracy. Optimal registration marks achieving this bound are discussed. These optimal designs, however, require very high printing or etching resolution and are inherently very sensitive to variations in the image sampling model (like scaling of grid size and rotation). More robust, optimal and suboptimal “topology-preserving” registration marks are then introduced and analyzed.

Index Terms—Grid geometry, image registration, information-theoretic precision bounds.

I. INTRODUCTION

Suppose that a planar shape is digitized by point sampling at lattice points defined by a square grid. The result is a binary two-dimensional “digital image” of the shape; a pattern of zeros and ones indicating whether the corresponding grid point belongs to the shape or its background (see Fig. 1). In case the planar shape is known up to an arbitrary translation in the plane, the two-dimensional pattern of zeros and ones that form its digital image provides information about its location in the plane. Formally, the planar shape \(S\) can be described by an indicator function over \(\mathbb{R}^2\)

\[
\chi(x,y) = \begin{cases} 
1, & \text{if } (x, y) \in S \\
0, & \text{if } (x, y) \notin S.
\end{cases}
\]  

(1.1)

If the shape is translated by a vector \((X, Y)\), the translated shape \(S(X, Y)\) has an indicator function given by

\[
\chi_{S(X, Y)}(x, y) = \chi(x - X, y - Y).
\]  

(1.2)

We digitize translated versions of \(S\) on the unit grid \(\{(x, y) \in \mathbb{Z}^2\}\). The result of digitizing \(S(X, Y)\), on the lattice is

\[
B(i, j) = \chi_{S(X, Y)}(i, j) = \chi(1 - X, 1 - Y).
\]  

(1.3)

We address in this correspondence the following questions:
1) Given a planar shape of finite support, say \(S \subset \mathbb{Z}^2 \times \mathbb{Z}^2 \times \mathbb{Z}^2 \subset B^2 \) (i.e., \(\chi(x, y) = 0 \) for \((x, y) \notin B^2 \)), how should we estimate the translation vector \((X, Y)\) from \(B(i, j)\)?
2) What is the best that we can do in estimating the location of \(S\) over all the possible planar shapes of finite extent and what shapes achieve minimum error in location?
3) How to design “good” shapes for location estimation when the shapes are constrained to obey certain further restrictions on size, topology, etc.

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A Comparison of Fiducial Shapes for Machine Vision Registration

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ABSTRACT

One way to perform registration and alignment for machine assembly is with respect to precisely located landmarks, called fiducials, that are located by machine vision means. For applications such as electronics assembly, where densities are high and tolerances must be low, the precision by which the fiducials are located affects everything aligned relative to them. Because of spatial sampling effects, differently shaped fiducials can be measured with different levels of precision. In past work we have determined and compared the minimax precision error for simple geometric shapes, and extended the results to propose a concentric pattern as having desirable qualities of high location precision and rotational invariance. We reiterate this work, and extend it to examine the performance of the concentric fiducial, as a function of diameter, number of rings, noise, and ring spacing.

1. Introduction

Electronics assembly, robotics manipulation, and many other manufacturing applications, require precise registration to assure proper positioning and alignment. One way to perform registration is to position everything with respect to one or more landmarks, called fiducial marks, or simply fiducials. For the electronics application, fiducials are positioned in precise and known locations relative to circuit traces. Then registration is performed relative only to the fiducials, independent of any imprecision of absolute positioning on the machine. In this paper, we reiterate past work [1], on determining the minimax precision of simple geometrically shaped fiducials, and extend our examination of a concentric pattern, which was proposed in [2] as having desirable fiducial characteristics.

In this paper, location is measured by the centroid calculation, because this measurement is performed on spatially sampled data, there may be a difference between the true (unsampled) centroid location and that measured from the pixels. The Euclidean distance, measured in units of pixels, between the true and measured centroid of a fiducial is called the precision error. There are other methods for determining location, which are reviewed in reference [2], but the centroid calculation has an advantage that it is simple and fast. Besides work on registration, there is also a body of work in the area of subpixel precision that is applicable to this paper. Many of these papers have also been referenced in [2] but for completeness we mention reference [3] on imprecision regions (called "locales") due to spatial sampling, references [4–5] on digital discs and rings, and a more recent paper [6], dealing with the effects of noise on locale shape and size.

2. Shape and Size of Simple Geometric Fiducials

In an earlier work [1] the subpixel registration precision of simple geometrically shaped fiducials was studied. Using analysis and experiment, the maximum error in the centroid due to spatial sampling was examined for different shapes and parameters. For completeness, we summarize this work here.

For purposes of analysis and experiment, the image is assumed to be binary. The binary images are created by assigning a 1 to a pixel \( p(x, y) \) if its center is found to be within the analog fiducial region, and 0 otherwise. For determination of the effects of sampling, the center of the concentric fiducial was shifted uniformly within \((-0.5, 0.5)\) to increments of 0.01 pixels in \( x \) and \( y \). The maximum of the errors for all shifts within this region is found and recorded.

To test the effects of size, a dimension of the fiducial is incremented in 0.25 pixel steps over a range of 2 to 22 pixels, and the change of error is examined.

Results in Figure 1 show the precision error plotted against the size (sidelength for square, vertical diagonal for diamond with diagonal fixed, and diameter for circle). It can be seen here, and is explained in more detail in references [1–2], that, while the precision error for the square is at best, 0.25 pixels, both the diamond and circle have errors that decrease with larger sizes. While none of the plots decrease monotonically, that of the diamond has very large deviations from minimum whereas the circle has smaller local maxima. This non-monotonicity in the decrease of the error curve is due to the spatial sampling effects between the Cartesian grid and the continuous shape. For the diamond shape, rotation of a fixed-size shape produces similar effects. In the following sections, we choose to develop and expand upon use of the circular shape as a fiducial because of its relatively small and low-deviation error as shown in Figure 1, and because its shape is rotationally invariant.

3. Centroid Calculations and the Concentric Fiducial

In this section, we review the development from circular to concentric fiducial. This development is made by modifying the centroid calculation method. The most straightforward method of centroid determination is just to find the average of the 1-pixel locations, \( \left( M_x, M_y \right) \):

\[
M_x = \frac{1}{A} \sum_{y} \sum_{x} x p(x,y) \quad M_y = \frac{1}{A} \sum_{y} \sum_{x} y p(x,y)
\]

where \( A = \sum_{y} \sum_{x} p(x,y) \quad p(x,y) = \{0,1\} \). Knowledge of the fiducial size and shape can be exploited to improve the measure. Consider that it is not necessary to sum all the pixels within the fiducial; instead, with knowledge that the disk is filled (completely 1-valued), the edges can be found, and the same centroid calculated just from these edge locations. For the edges of a square starting at \( x_0, y_0 \) and ending at \( x_0, y_0 \) for rows of \( y \), the centroid and area can be calculated:
Subpixel Image Registration Using Circular Fiducials

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ABSTRACT

The design of fiducials for precise image registration is of major practical importance in computer vision, especially in automatic inspection applications. We analyze the subpixel registration accuracy that can, and cannot, be achieved by some rotation-invariant fiducials, and present and analyze efficient algorithms for the registration procedure. We rely on some old and new results from lattice geometry and number theory and efficient computational-geometric methods.

Keywords: Computer Vision, Registration, Fiducials, Lattice Geometry, Number Theory.

1. Introduction

The design of fiducials for accurate registration of images is a problem of major practical importance in computer vision. Imagine the following scenario: An electronic printed circuit board is to be automatically inspected for manufacturing defects. The system that does this is a computer equipped with a digital camera. A binary image of the board, obtained via the camera, is the input to the inspection algorithm, which then checks select areas of the image for the defects. The algorithm must first locate accurately those areas of the board in the image plane. To enable accurate location of points on the board, special patterns ("fiducials") imprinted on the board are located in the image, and a select point in each fiduciial serves as an "anchor" point, relative to which other points are referenced. Usually

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Optimum Fiducials Under Weak Perspective Projection

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Abstract. We investigate how a given fixed number of points should be located in space so that the pose of a camera viewing them from unknown locations can be estimated with the greatest accuracy. We show that optimum solutions are obtained when the points form concentric complete regular polyhedra. For the case of optimal configurations we provide a worst-case error analysis and use it to analyze the effects of weak perspective approximation to true perspective viewing. Comprehensive computer simulations validate the theoretical results.

Keywords: pose recovery, weak perspective projection, fiducial design

1. Introduction

In many applications, there is a need to locate and track objects in space from sequences of images provided by a calibrated camera. In robotics one is faced with the dual problem of self location given images of objects having known 3-D positions. To accomplish these goals, it is often the case that we can affix labels to the objects of interest to generate point features that can be easily located in the images acquired. In this paper we analyze the problem of designing point constellations for optimal recovery of space location and orientation. These point constellations, or 3-D fiducial objects, should then be deployed in the environment, or affixed to the objects that are to be tracked. The viewing transformation is assumed to be weak perspective, which involves an orthographic projection and a scaling, and both noise and quantization are assumed to further distort the images acquired. Therefore it makes sense to ask the question, “how should we design 3-D point patterns or constellations that best exploit the volume allocated to them in the sense of providing the most precise 3-Dimensional location and orientation information when viewed from arbitrary (but sufficiently distant) places to justify the weak perspective assumption?”

The paper starts with an analysis of methods to recover orientation and translation of known constellations of points viewed by a weak perspective camera. The basic framework is known and has been the subject of investigation by Huang and Lee (1989) who provided a linear algorithm for motion/structure recovery from three views of four points, Tomasi and Kanade (1992), who studied recovery of both 3-D structure and motion for feature point constellations viewed by a moving camera, and by Ullman and Basri (1991), who dealt with the problem of recognizing novel views of such point constellations from a small set of views considered as models. We assume not only that the point
New Devices for 3D Pose Estimation: Mantis Eyes, Agam Paintings, Sundials, and Other Space Fiducials

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Abstract. Several unconventional ideas for viewer/camera pose estimation are discussed. The methods proposed so far advocate the use of advanced image processing for identification and precise location of calibration objects in the images acquired, and base pose recovery on the identification of the viewing dependent deformations of these objects. We propose to more fully exploit the freedom in the design of "space fiducials" or calibration objects showing that we can build objects whose images directly encode, in easily identifiable gray-level/color or temporal patterns, the pose of their viewer. We also show how to construct high-precision fiducials, which can determine a viewing direction quite accurately when it is known to lie within a relatively narrow range.

Keywords: Pose determination, space fiducials

1. Introduction

What you see tells you a lot about where you are. People know this and over the years have employed various methods to estimate the pose (i.e. position and orientation) by exploiting images that cameras acquire. Usually it is assumed that in the field of view of the camera there are some feature points (as determined by edges and corners of known objects or on specially designed calibration images), whose relative geometry is a-priori known. If we know the geometry of the feature points and can uniquely identify them in a perspective image taken by a camera whose intrinsic parameters are known, then pose estimation becomes a classical problem of photogrammetry and computer vision. Numerous versions of this problem involving different numbers of feature points, arranged in various spatial patterns, have already been addressed in the literature. Research was also devoted to pose estimation based on features other than points, like, for example, lines and circles/conic sections or even solid objects such as cubes or planar images displaying various shapes. In all cases, however, pose estimation was addressed by first determining the precise location (via sophisticated image analysis algorithms) of the features whose intrinsic geometry was assumed to be a-priori known, and thereby reducing the problem to the equivalent "algebraic" framework of feature points whose precise perspective projection is provided by the images seen (see, e.g., Basu, 1995; Ganapathy, 1984; Haralick et al., 1994; Holt and Netravali, 1991; Robert, 1996; Tsai, 1987; Bruckstein et al., 1999).

The algebraic framework is very nice indeed and it provides the possibility to do the pose recovery computations that involve solving nonlinear (but polynomial)

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Optimized Overlay Metrology Marks: Theory and Experiment

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Abstract—In this paper, we provide a detailed analysis of overlay metrology mark and find the mapping between various properties of mark patterns and the expected dynamic precision and fidelity of measurements. We formulate the optimality criteria and suggest an optimal overlay mark design in the sense of minimizing the Cramer–Rao lower bound on the estimation error. Based on the developed theoretical results, a new overlay mark family is proposed—the grating marks. A thorough testing performed on the new grating marks shows a strong correlation with the underlying theory and demonstrates the superior quality of the new design over the overlay patterns used today.

Index Terms—box-in-box marks, Cramer–Rao lower bound, dynamic precision, Fisher information matrix, grating marks, overlay mark, overlay mark fidelity, overlay metrology.

I. INTRODUCTION

A CCURATE and precise overlay metrology is a critical requirement in order to achieve high product yield in microelectronic manufacturing. New challenges become evident as micro lithography processes are developed for each new design rule node. A critical link in the overlay metrology chain is the metrology mark which is chosen to be included on the reticle, printed on the wafer, subsequently processed and which is ultimately imaged in the metrology tool in the metrology process. In this publication a theoretical and experimental study is described which sheds new light on the limitations of existing mark designs while proposing and validating new designs of superior performance.

In Fig. 1 a standard overlay (BiB) mark is shown schematically. It consists of two “boxes” printed on two subsequent layers—top (grey) and bottom (black)—between which the overlay is measured. By design the centers of symmetry of the inner (grey) and outer (black) boxes coincide. The actual overlay appears as misregistration between the centers of symmetry of the “black” and “grey” layers.

There are two major use cases in overlay metrology for micro lithography. The first and the most obvious is termed lot dispositioning. If measured overlay exceeds some allowable threshold, the lot cannot proceed to the next process step. This generally results in rework, that is the lot is returned to the previous lithography step after the resist is stripped. This is provided the overlay measurements were done immediately after development. Under some circumstances the overlay measurements after development are not viable, and are done after etch. In this case, there is no option for rework, and lots outside of allowable thresholds are scrapped.

The second use case of overlay metrology is for correction of the exposure tool. Usually, the overlay is measured at four corners of the field and over several fields on the wafer, which provides the necessary statistical sampling to enable stepper corrections model to be calculated. This model includes intra-field and inter-field correctibles, such as offset, rotation and scale. These correctibles are fed back to the exposure tool to improve performance on subsequent lots.

Conventional BiB based metrology has been the standard overlay metrology for almost two decades. However, as the overlay budget shrinks together with the lithographic design rules, a number of performance limitations are becoming evident. These shortcomings are addressed in the section below. Application of grating structures to lithography and metrology fields is being extensively studied. One of such applications
Iterative Algorithm for Optimal Fiducials Under Weak Perspective Projection

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ABSTRACT: In previous work, we designed space fiducials with the aim of making camera pose determination as noise-intensive as possible. These fiducials turned out to be sets of points that formed concentric regular polyhedra. Here, we apply an idea of Dementhon and Davis and test and analyze an iterative linear algorithm in conjunction with our optimal fiducials to increase the accuracy of the computed camera pose. We also analyze under what circumstances this iterative algorithm is guaranteed to converge to the correct solution. Comprehensive computer simulations illustrate the behavior of the algorithm and the degree of improvement in pose determination in case of convergence. © 2009 Wiley Periodicals, Inc Int J Imaging Syst Technol, 19, 27–36, 2009; Published online In Wiley InterScience (www.interscience.wiley.com); DOI 10.1002/ima.20175

Key words: Iterative algorithm; fiducials; weak perspective; computer vision

I. INTRODUCTION

In Bruckstein et al. (1999), we investigated the problem of deciding where a given fixed number of points in space should be located so that the pose of a camera viewing them from unknown locations can be analyzed with the greatest accuracy. Under the assumption that image points were obtained by weak perspective projection, we found that the optimal point configurations formed concentric regular polyhedra. In the process of drawing this conclusion we used a straightforward matrix inversion based algorithm to calculate the camera pose.

In this article we consider an iterative pose recovery algorithm of Dementhon and Davis (1995) and adapt it to our setting of optimal fiducials to improve pose recovery performance in regions where the weak perspective projection is a poor approximation to the true perspective. This happens when the object is near the camera, in particular when the distance from the object to the camera is less than 20 times the diameter of the object. This iterative algo-

rithm starts with the weak perspective assumption for pose recovery and then successively improves the estimates of pose by using the current estimate of the 3D structure to shift the position of feature points in the image plane towards what would be their "correct" weak perspective projection. We note that the algorithm of Dementhon and Davis, (1995) was further analyzed and improved by Horn et al. (1997), incorporating puncture perspective approximation in the iteration process. Also, the Dementhon and Davis algorithm was modified in a different manner by Chung and Tsai, (2002) in developing a technique for determining facial pose and expression.

In this article, we first analyze under what circumstances the algorithm is guaranteed to converge monotonically to the correct solution, and then present simulation results showing that the iterative pose recovery process works very well, even beyond the monotone convergence region.

II. FORMULATION

For purposes of fiducial design for efficient pose recovery we assume that we can designate where we place several feature points \(P_i = [X_i, Y_i, Z_i]^T\) in the environment. Hence, we assume that the coordinates of the \(P_i\) are known in the world coordinate system. These points may be observed from a camera in any position, and thus the camera coordinate system is related to the world coordinate system by an arbitrary rotation \(R\) and translation \(T\). A point \(P'_i = [x'_i, y'_i, z'_i]^T\) in the camera system corresponding to \(P_i\) in the world system thereby satisfies

\[
P'_i = RP_i + T,
\]

with

\[
R = \begin{bmatrix}
  r_1 & r_2 & r_3 \\
r_4 & r_5 & r_6 \\
r_7 & r_8 & r_9
\end{bmatrix}
\]

and

\[
T = \begin{bmatrix}
r_1 \\
r_2 \\
r_3
\end{bmatrix}.
\]

We let \(p'_i = [x'_i, y'_i]^T\) denote the corresponding image point in the image plane of the camera. Under true perspective viewing with a camera having a focal length \(f\), the projection equations are
Chapter 1
Digital Geometry in Image-Based Metrology

Alfred M. Bruckstein

Abstract Interesting issues in digital geometry arise due to the need to perform accurate automated measurements on objects that are "seen through the eyes" of modern imaging devices. These devices are typically regular arrays of light sensors and they yield matrices of quantized probings of the objects being looked at. In this setting, the natural questions that may be posed are: how can we locate and recognize instances from classes of possible objects, and how precisely can we measure various geometric properties of the objects of interest, how accurately can we locate them given the limitations imposed upon us by the geometry of the sensor lattices and the quantization and noise omnipresent in the sensing process. Another interesting area of investigation is the design of classes of objects that enable optimal exploitation of the imaging device capabilities, in the sense of yielding the most accurate measurements possible.

1.1 Introduction

Scanned character recognition systems are by now working quite well, several companies emerged based on the need to do image based inspection for quality control in the semiconductor industry and, in general, automated visual inspection systems are by now widely used in many areas of manufacturing. In these important applications one often needs to perform precise geometric measurements based on images of various types of planar objects or shapes. Images of these shapes are provided by sensors with limited capabilities. These sensors are spatially arranged in regular planar arrays providing matrices of quantized pixel-values that need to be processed by automated metrology systems to extract information on the location, identity, size and orientation, texture and color of the objects being looked at. The geometry, spatial resolution and sensitivity of the sensor array are crucial factors in the measurement performances that are possible. When sensor arrays are regular planar grids, we have to deal with a wealth of issues involving geometry on the integer grid,
Simple and Robust Binary Self-Location Patterns

Alfred M. Bruckstein, Tuvi Etzion, Fellow, IEEE, Raja Giryes, Noam Gordon, Robert J. Holt, and Doron Shuldiner

Abstract—A simple method to generate a 2-D binary grid pattern, which allows for absolute and accurate self-location in a finite planar region, is proposed. The pattern encodes position information in a local way so that reading a small number of its black or white pixels at any place provides sufficient data from which the location can be decoded efficiently and robustly.

Index Terms—de Bruijn sequences, M-sequences, self-location patterns.

I. INTRODUCTION

TAKE a blindfolded man on a random one-hour walk around town and then remove his blindfold. How will he know where he is? He has several options, based on the information he can gather. The man could carefully count his steps and take note of every turn during the blindfolded walk to know his location relative to the beginning of his trip. Armed with a navigation tool such as a sextant or GPS unit, he could ask the stars or the GPS satellites where he is. Finally, he could simply look around for a reference, such as a street sign, a landmark building, or even a city map with a little arrow saying “You are here.”

There are numerous applications where a similar problem is encountered. We need to somehow position the mobile devices used in everyday life, using some sort of sensory input. Wheeled vehicles can count the turns of their wheels much like the man counting his steps. Similarly, many devices, from industrial machine stages to ball-mice, employ sensors that are coupled with the mechanics and count small physical steps of a known length, in one or more dimensions. The small relative position differences can be accumulated to achieve relative self-location to a known starting point. More recent technologies, such as those found in modern optical mice, use imaging sensors instead of mechanical encoders to estimate the relative motion by constantly inspecting the moving texture or pattern of the platform beneath them.

Sometimes the inherent accumulating error in relative self-location methods, or some other reasons, make them infeasible or unfit for certain applications, where we would want the capability to obtain instant and accurate absolute self-location. Given several visible landmarks of known locations, a mobile robot could calculate its position through a triangulation [1]. Alternatively, cleverly designed space fiducials (e.g., [2]), whose appearance changes with the angle of observation, can also serve for self-location.

Much like street signs for people, there are absolute self-location methods that provide sufficient local information to the device sensors, such that the absolute positioning can be attained. Specifically, planar patterns have been suggested, where a small local sample from anywhere in the pattern provides sufficient information for decoding the absolute position. A naive example would consist of a floor filled with densely packed miniature markings, in which the exact coordinates are literally inscribed inside each marking. Of course, that would require a high sensor resolution and color recognition capabilities. Indeed, there are much more efficient methods, which do with considerably less geometric detail in the pattern. Some commercial products have been utilizing these approaches, e.g., a pen with a small imaging device in its tip, writing on paper with a special pattern printed on it, which allows full tracking of the pen position at any time [3].

A classic method for absolute self-location in 1-D is the use of de Bruijn sequences [4], [5]. A de Bruijn sequence of order \( n \) over a given alphabet of size \( q \) is a cyclic sequence of length \( q^n \), which has the property that every possible sequence of length \( n \) of the given alphabet appears in it as a consecutive subsequence exactly once. Thus, sampling \( n \) consecutive letters somewhere in the sequence is sufficient for perfect positioning of the sampled subsequence within the sequence. Several methods for the construction of de Bruijn sequences have been proposed, e.g., [5]–[8]. There is also a 2-D generalization, i.e., it is possible to construct a 2-D cyclic array in which each rectangular subarray of a certain size \( k \times n \) appears exactly once in the array. These types of arrays are called perfect maps, e.g., [9] and [10], and they can serve as the basis for absolute self-location on the plane.

Of special interest and importance in communication are maximal-length linear shift-register sequences known also as M-sequences or pseudonoise sequences [11]. An \( M \)-sequence of order \( n \) is a sequence of length \( 2^n - 1 \) generated by a linear feedback shift-register of length \( n \). In a cyclic sequence of this type, each nonzero \( n \)-tuple appears as a window of length \( n \).
RUNE-Tag: a High Accuracy Fiducial Marker with Strong Occlusion Resilience

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Abstract

Over the last decades fiducial markers have provided widely adopted tools to add reliable model-based features into an otherwise general scene. Given their central role in many computer vision tasks, countless different solutions have been proposed in the literature. Some designs are focused on the accuracy of the recovered camera pose with respect to the tag; some other concentrate on reaching high detection speed or on recognizing a large number of distinct markers in the scene. In such a crowded area both the researcher and the practitioner are licensed to wonder if there is any need to introduce yet another approach. Nevertheless, with this paper, we would like to present a general purpose fiducial marker system that can be deemed to add some valuable features to the pack. Specifically, by exploiting the projective properties of a circular set of sizeable dots, we propose a detection algorithm that is highly accurate. Further, applying a dot pattern scheme derived from error-correcting codes, allows for robustness with respect to very large occlusions. In addition, the design of the marker itself is flexible enough to accommodate different requirements in term of pose accuracy and number of patterns. The overall performance of the marker system is evaluated in an extensive experimental section, where a comparison with a well-known baseline technique is presented.

1. Introduction

A fiducial marker is, in its broadest definition, any artificial object consistent with a known model that is placed in a scene. At the current state-of-the-art such artifacts are still only choice whenever a high level of precision and repeatability in image-based measurement is required. This, for instance, the case with accurate camera pose estimation, 3D structure-from-motion or, more in general, any flavor of vision-driven dimensional assessment task. Of course a deluge of approaches have been proposed in order to obtain a reasonable performance by relying only on natural features already present in the scene. To this extent, several repeatable and distinctive interest point detection and matching techniques have been proposed over the years. While in some scenarios such approaches can obtain satisfactory results, they still suffer from shortcomings that severely hinder their broader use. Specifically, the lack of a well-known model limits their usefulness in pose estimation and, even when such a model can be inferred (for instance by using bundle adjustment) its accuracy heavily depends on the correctness of localization and matching. Moreover, the availability and quality of natural features in a scene is not guaranteed in general. Indeed, the surface smoothness found in most man-made objects can easily lead to scenes that are very poor in features. Finally, photometric inconsistencies due to reflective or translucent materials jeopardizes the repeatability of the detected points. For this reasons, it is not surprising that artificial fiducial tags continue to be widely used and are still an active research topic. For practical purposes, most markers are crafted in such a way as to be easily detected and recognized in images produced by a pinhole-modeled camera. In this sense, their design leverages the projective invariants that characterizes geometrical entities such as lines, planes and conics. It is reasonable to believe that circular dots were among the first shapes used. In fact, circles appear as ellipses under projective transformations and the associated conic is invariant with respect to the point of view of the camera. This allows both for an easy detection and a quite straightforward rectification of the circle plane. In his seminal work Guttell [7] proposes to use a set of highly contrasted concentric circles and to validate a candidate marker by exploiting the compatibility between the centroids of the ellipses found. By alternating white and black circles a few bits of information can be encoded in the marker itself. In [3] the concentric circle approach is enhanced by adding colors and multiple scales. In [11] and [16] dedicated “data rings” are added to the fiducial design. A set of four circles located at the corner of a square is adopted by [4]: in this case an identification pattern is placed at the centroid of the four dots in order to distinguish between different targets. This ability to recognize the viewed markers is very important
Designing Highly Reliable Fiducial Markers

Mark Fiala, Mamber, IEEE

Abstract—Fiducial markers are artificial landmarks added to a scene to facilitate locating point correspondences between images, or between images and a known model. Reliable fiducials solve the interest point detection and matching problems when adding markers is convenient. The proper design of fiducials and the associated computer vision algorithms to detect them can enable accurate pose detection for applications ranging from augmented reality, input devices for HCl, to robot navigation. Marker systems typically have two stages; hypothesis generation from unique image features and verification/identification. A set of criteria for high robustness and practical use are identified and then optimized to produce the ARTag fiducial marker system. An edge-based method robust to lighting and partial occlusion is used for the hypothesis stage, and a reliable digital coding system is used for the identification and verification stage. Using these design criteria large gains in performance are achieved by ARTag over conventional ad hoc designs.

Index Terms—Augmented reality, fiducial marker systems, computer vision.

1 INTRODUCTION

Fiducial markers are useful in many situations where object recognition or pose determination is needed with a high reliability, where natural features are not present in sufficient quantity and uniqueness, and where it is not convenient to affix markers. Example applications include indoor augmented reality, handheld objects for user pose input, message logs to trigger a behavior, or generic pose determination in industrial settings. Despite future improvement in markerless computer vision, there will probably always be applications such as featureless indoor scenes where adding markers is superior.

Marker design is often performed ad hoc due to an absence in the literature of adequate analytical approaches. With proper design markers can be made to be very reliable and outlier detections or matches can be reduced to occurring with very low probabilities. The ARTag fiducial marker system [1], [2] was designed with the approaches detailed herein. Specifically, digital coding techniques for the verification and identification stage are described. Several augmented reality software development kits use ARTag for reliable pose determination in cluttered scenes despite uncontrolled lighting.

A fiducial marker system consists of some unique patterns along with the algorithms necessary to locate their projection in camera images. Such a system should reliably report one or more points per marker when seen in the image. The patterns should be distinct enough not to be confused with the environment. Ideally, the system should have a library of many unique markers that can be distinguished one from another. The image processing should be robust enough to find the markers in situations of uncontrolled lighting, image noise and blurring, unknown scale, and partial occlusion. Preferably, the markers should be passive (not requiring electrical power) planar patterns for convenient printing and mounting, and should be detectable with a minimum of required image pixels to maximize the range of usage.

The most common markers used are constellations of circular dots or colored markings. In recent years, many augmented reality (AR) projects have used ARTag [3] marker, which consist of a square outline with an internal pattern identified by correlation. The ARTag marker system is a more recent system gaining popularity in AR projects due to its improved performance.

A fiducial marker system is designed to solve the following problem: Given an input image (either a static image or a frame from a video stream), provide a list of markers found in the image. Fig. 1 shows ARTag patterns being recognized in an image. This extracted information can be used in different ways, the simplest being to use only the marker ID to trigger some behavior dependent only on the presence of a marker such as logging into a kiosk. The image position information can also be used, which makes it possible to use a marker or array of markers as a handheld computer mouse pointer. Finding pose from one or more markers finds use in applications such as augmented reality where the camera pose is used to align a virtual camera with the real one to overlay virtual objects (Fig. 1b).

The paper is organized as follows: Criteria are identified to evaluate a marker system, a number of bar code, and planar marker systems are introduced. The ARTag marker system and its design according to the criteria is described, a general design methodology for digital marker codes is given, and, finally, some applications are presented that are only possible with a highly reliable system created using the approaches given herein.

2 MEASURING FIDUCIAL MARKER SYSTEM PERFORMANCE

A fiducial marker system can be rated by how reliably it finds image points matching physical points on the markers. Failure modes typically include poor lighting, unusual lighting conditions and falsely reporting objects similar in appearance to markers in cluttered scenes. A robust marker system will provide such false outlier measurements, which may complicate a system design. The shortcomings of a tracking system are readily apparent in Augmented Reality applications when augmentations do not register well with real objects, if they flicker or vibrate, or if the wrong virtual object appears. The robustness and usefulness of a fiducial marker system can be characterized by some numerical metrics and some careful qualitative observations. The former, if properly stated, give quantities to optimize. Eleven practical evaluation criteria, which other fiducial marker systems do not fully address:

1. The false positive rate,
2. The intermarker confusion rate,
3. The false negative rate,
4. The minimal marker size,
5. The vertex filter characteristics,
6. The marker library size,
7. Immunity to lighting conditions,
8. Immunity to occlusion,
9. Perspective support,
10. Immunity to photometric calibration,
11. The speed performance.

All but number seven or eight can be determined theoretically or with experimentation. The failure to properly address any of these 11 criteria greatly reduces the usability of a marker system.

The false positive rate is the rate of falsely reporting the presence of a marker when none is present. The intermarker confusion rate is the rate of when the wrong id is reported, i.e., one marker was mistaken for another. The false negative rate is the probability that a marker is present in an image but not reported. Another metric is criterion 4, the minimal marker size, which is the size in pixels required for reliable detection. The smaller the marker needs to be

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Recommended for acceptance by Y. Sato.
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The Geometry of Colorful, Lenticular Fiducial Markers

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Abstract

Understanding the pose of an object is fundamental to a variety of visual tasks, from trajectory estimation of UAVs to object tracking for augmented reality. Fiducial markers are visual targets designed to simplify this process by being easy to detect, recognize, and track. They are often based on features that are partially invariant to lighting, pose and scale. Here we explore the opposite approach and design passive calibration patterns that explicitly change appearance as a function of pose. We propose a new, simple fiducial marker made with a small lenticular array, which changes its apparent color based on the angle at which it is viewed. This allows full six-degree-of-freedom pose estimation with just two markers and an optimization that fully decouples the estimate of rotation from translation. We derive the geometric constraints that these fiducial markers provide, and show improved pose estimation performance over standard markers through experiments with a physical prototype for form factors that are not well supported by standard markers (such as long skinny objects). In addition, we experimentally evaluate heuristics and optimizations that give robustness to real-world lighting variations.

1. Introduction

Many visual applications are based on tracking the pose of an object relative to a camera. Often, fiducial markers are attached to those objects to simplify this pose estimation process. These markers are designed to be easy to detect and to track, for example small spheres that look similar from any viewpoint. In this paper, we consider an alternative in fiducial marker design, creating markers whose colors change based on their relative pose to the camera.

There are many possibilities for non-lambertian materials that might support this process, from active electronics to holographic materials, but we would like to select for three properties. First, we would like something easy to create from available materials. Second, we would like a passive marker that does not require power. Third, we would like a clear geometric interpretation to the appearance.

Lenticular sheets are available in hobby shops and often used to create children's toys, bookmarks, or promotional material that show an animated pattern as they turn. The optical properties of the lenticular plastic use ambient light to show different patterns when viewed from different directions. In this paper we describe how to use lenticular printing to create fiducial markers. Figure 1 illustrates the process. We design a pattern that can be printed on standard color laser printer. Adhering this to the back of a plastic lenticular sheet creates small markers whose color relates to their orientation. The contributions of this paper are:

1. The description and evaluation of a cheap way to create fiducial markers that explicitly change their apparent color based on their orientation relative to the camera.
2. The derivation of geometric constraints relating the location and color of lenticular fiducial markers to object pose.

Figure 1: Gluing a lenticular plastic sheet (left) to a color pattern (center) from a standard color printer is a convenient source of fiducial markers (right) whose apparent color has a clear geometric relationship to the relative orientation to the camera. Several small markers are shown leaning against a nickel and they appear to be different colors because they are oriented differently relative to the camera. This paper discusses the design of these markers and how to use them as fiducial markers for pose estimation.
A Motion Blur Resilient Fiducial For Quadcopter Imaging

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Abstract

Fiducials are commonly placed in environments to provide a uniquely identifiable object in the scene. In quadcopter applications, these fiducials are often used to evaluate planning algorithms given that ground truth positions can be detected from the quadcopter's camera.

Low cost quadcopters, however, are subject to quick and unstable motions that can cause significant motion blur that severely affects the detection rate of existing fiducials. This problem motivated us to design a fiducial that is robust to motion blur. Our proposed design uses concentric circles with the observation that the direction perpendicular to the motion blur direction will be relatively unaffected by the blur. As a result, an appropriate fiducial code orthogonal to the blur direction can be recognized. Since the direction of motion blur is unknown, the circular design is good for all motion blur directions. We describe the design of binary fiducials, and also a detection algorithm. We show that our marker can significantly outperform existing fiducials in scenes captured with a quadcopter.

1. Introduction

The recent availability of low-cost quadcopters has helped fuel significant efforts in research focused on unmanned aerial vehicles. Navigation and planning of these vehicles is typically performed using onboard inertial sensors and/or vision based modules that uses visual cues in the real world. However, to evaluate the effectiveness of navigation methods, one or more fiducials are commonly placed [7, 18, 16] in the environment to provide additional information that serves for ground-truth positional measurements.

In order to be effective, fiducial markers (or simply, fiducials) need to be easily detected in the scene. A variety of fiducials have been proposed [20, 1, 13, 5, 4] in the literature. These take the form of binary codes arranged into rectangular grids [1, 13] or other geometric primitives arranged in predefined spatial patterns [20, 5, 4]. Figure 1 shows the popular ARTag [13] fiducial as possibly seen from a quadcopter. A problem for existing fiducials is that low-cost quadcopters often exhibit very quick and erratic physical movements that result in motion blur evidenced in the images captured from the quadcopter's onboard camera. This motion blur has an adverse effect on the recognition of fiducial markers. This can be seen in Figure 1–(b) where the ARTag fiducial cannot be recognized due to motion blur. This is not too surprising as most existing fiducials are not designed to handle motion blur.

Compounding this problem is the additional issue of dropped video frames from the quadcopter’s wireless communication module. This means that not only is blur a problem, but there may be large discontinuities in the pattern’s position due to missing video frames. As we will show in Section 4, this later problem makes it challenging to apply tracking algorithms that can exploit temporal coherence for determining the fiducial’s position.

Contributions: To address these problems, we propose a fiducial that is designed to be resistant to motion blur. Our design is based on circles as shown in Figure 1–(c,d). The design is based on the observation that motion blur from a quadcopter tends to be linear in nature. As such, when our fiducial is blurred, there is no blur in the direction perpendicular to the direction of motion. This allows the signature of the fiducial to remain intact in any direction.

When multiple fiducials are present, using concentric rings, we can treat the presence or absence of a ring as a bit, allowing us to assign a code to the marker. Our experiments show that these designs can significantly outperform existing codes under the presence of motion blur. As far as we are aware, this is the first work to propose a blur resistant marker especially in quadcopter settings.

The remainder of this paper is organized as follows. Section 2 gives an overview of related work in fiducials, as well as the related problem of tracking. Section 3 motivates our fiducial design by analyzing the performance of existing codes under motion blur, and describes our detection algorithm. Section 4 shows several experiments using quadcopter imagery. This is followed by a discussion and a summary in Section 5 and Section 6 respectively.
Detection and Accurate Localization of Circular Fiducials under Highly Challenging Conditions

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Abstract

Using fiducial markers ensures reliable detection and identification of planar features in images. Fiducials are used in a wide range of applications, especially when a reliable visual reference is needed, e.g., to track the camera in cluttered or textureless environments. A marker designed for such applications must be robust to partial occlusions, varying distances and angles of view, and fast camera motions. In this paper, we present a robust, highly accurate fiducial system, whose markers consist of concentric rings, along with its theoretical foundations. Relying on projective properties, it allows to robustly localize the imaged marker and to accurately detect the position of the image of the (common) circle center. We demonstrate that our system can detect and accurately localize these circular fiducials under very challenging conditions and the experimental results reveal that it outperforms other recent fiducial systems.

1. Introduction

The term fiducial marker, or simply fiducial, refers to a set of (coplanar) points encoded in a planar pattern allowing a reliable detection and identification across views. A fiducial marker system is a set of fiducial marker(s) coupled with dedicated computer vision algorithms solving the detection and identification problems. This is used in a variety of applications, both in computer vision and robotics, ranging from camera calibration to augmented reality or visual SLAM. The choice of fiducials is of crucial importance within this framework as markers must provide reliable visual references in the scene that can be used to estimate, e.g., the camera position or its motion. Such framework requires that the fiducial marker system be robustly and accurately detectable even under very challenging conditions, such as, e.g., when the markers are partially or largely occluded, or seen under highly skewed angles or from long distances, when the illumination is very poor or irregular, or when the camera undergoes very fast motions generating blur.

In this paper, we present a robust, highly accurate and theoretically-founded fiducial system, which is highly tolerant to all of the mentioned challenges, as shown in Figure 1. Its markers are based on concentric black rings on a white background, extending the one-ring markers introduced by Gatell et al. in [10]. The geometric properties of the concentric circles delivered by their edges are exploited to accurately detect the image of the circle common center, thus providing a highly reliable feature point that can be used for tracking and motion estimation. The thickness of the rings can be used to encode the information of the marker, typically a unique ID, thus providing a simple and reliable
Lithography aware overlay metrology target design method


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ChromaTag: A Colored Marker and Fast Detection Algorithm

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Abstract

Current fiducial marker detection algorithms rely on marker IDs for false positive rejection. Time is wasted on potential detections that will eventually be rejected as false positives. We introduce ChromaTag, a fiducial marker and detection algorithm designed to use opponent colors to limit and quickly reject initial false detections and grayscale for precise localization. Through experiments, we show that ChromaTag is significantly faster than current fiducial markers while achieving similar or better detection accuracy. We also show how tag size and viewing direction effect detection accuracy. Our contribution is significant because fiducial markers are often used in real-time applications (e.g., marker assisted robot navigation) where heavy computation is required by other parts of the system.

1. Introduction

In this paper, we introduce ChromaTag, a new colored fiducial marker and detection algorithm that is significantly faster than current fiducial marker systems. Fiducial markers are artificial objects (typically paired with a detection algorithm) designed to be easily detected in an image from a variety of perspectives. They are widely used for augmented reality and robotics applications because they enable localization and landmark detection in featureless environments. However, because fiducial markers often complement large real-time systems (e.g., Camera Tracking [2, 9], SLAM [35] and Structure from Motion [8]), it is important that they run much faster than 30 frames per second. Figure 1 shows the run time of several state of the art markers, of which only ChromaTag achieves processing times significantly faster than 30 frames per second (all processing uses a 3.5 GHz Intel i7 Ivy Bridge processor).

ChromaTag achieves an orders-of-magnitude speedup with good detection performance through careful design of the tag and detection algorithm. Previous marker designs (Figure 2) typically use highly contrasting (black to white) borders [4, 7, 8, 10, 15, 24] for initial detection, but black-white edges are common in images and result in many initial false detections. IDs are decoded from the tags to verify detections, but decoding is the last step in the pipeline, so most of the time is spent rejecting false tags. Tags with distinctive color patterns can be used to limit initial false detections, but color consistency and the reduced spatial resolution of color channels (Bayer grid) create challenges for ID encoding and tag localization.

ChromaTag uses each channel of the LAB opponent colorspace to best effect. Large gradients between red and green in the A channel, which are rare in natural scenes, are used for initial detection. This results in few initial false detections that can be quickly rejected. The black-white border takes advantage of high resolution of the L channel for precise localization. The B channel is used to encode