Holographic Image Representations

**Problem:** Given an image $I(x,y)$ create a representation such that from any portion of the representation (contiguous or arbitrarily chosen) we can reconstruct an image $\hat{I}(x,y)$ with $d(I,\hat{I})$ decreasing with the size of $P$.

**Applications:**
- progressive refinement without order in packets of data
- coding resistant to loss of arbitrary portions of data
- anti-hierarchical schemes
Two Ideas for Holo-Representations

1. via Fourier Transforms

\[ I(x,y) \rightarrow I(x,y)e^{jP(x,y)} \]

\( P(x,y) \) random phase image
\( P(x,y) \) indep of \( P(x',y') \)
\( P(x,y) \sim \text{unif } [-\pi, \pi] \)

Then

\[ H(u,v) = \mathcal{F}^{-1}\{ I(x,y)e^{jP(x,y)} \} \]

is a holo-rep of \( I(x,y) \). Why? The random phase distributes the image information "evenly" in \( H(u,v) \).
Why does this work? [1-D explanation to keep it simple]

\[
\{I(k)\}_{k=0,1,\ldots,N-1} \rightarrow \{H(u) = \text{DFT}\{I(k)e^{j\pi TP(k)}\}\}
\]

If we define a window \( W(u) \):

\[
\begin{array}{c}
\text{a} \\
\text{a+L-1}
\end{array}
\]

and consider

\[
\hat{I}_w(k) = \text{DFT}\{H(u) \cdot W(u)\}
\]

we get that \( \hat{I}_w \) is a low pass filtered version of \( I(k)e^{j\pi TP(k)} \)

\[
\tilde{P}_a(k) = \{[P(k) + \frac{a+L-1}{L}k] \text{ mod } L\}
\]

are, due to \( P(k) \), i.i.d. \( U[0,1] \)

This makes all portions of length \( L \) equivalent!
Questions & Concerns:

- How good are the estimates of $I$ from portions of size $L$?

  We have

  $$E[|I_w|^2] \propto I^2 \quad \text{(factor } \frac{1}{M})$$

  variance $|I_w|^2 = \frac{1}{M} \sqrt{1-\frac{3}{M}} I^2$ if $L \leq \frac{M}{2}$

  $$= \frac{1}{M} \left[ \frac{M-L}{3M} + (M+L)^2 \right] I^2$$

  if $L > \frac{M}{2}$

  Noisy estimate if $L$ small, improving if $L \to M$.

- Do we need to know Window location?

  No! We can do DFT directly on portion of data available (i.e. without zero padding). No loss of quality, but location info necessary for progressive refinement.
• Do we increase the #bits representing the image?
  Indeed $H(u,v)$ is complex but we can code $(\text{Re } H, \text{Im } H)$ with 4÷6 bits each. So it is not necessarily the case that we need more bits. But clearly this is NOT a compression scheme!

• Isn't this scheme simply an implementation of computerized holography?
  Not according to our reading of the papers in digital holography.

• Shouldn't the same goal be achieved by a non transform domain method? see NEXT
2. Holographic Sampling of Images

Idea: $I(x, y) \rightarrow \text{"ordered" sequence of samples so that any portion of the sequence will contain samples of } I(x, y) \text{ as evenly spread in the image plane as possible.}$
4x4 and 8x8 examples

"Regular" holographic sampling

with rule:

Figure 3: Regular holographic sampling for a 4 × 4 image

Figure 4: Regular holographic sampling for an 8 × 8 image
64 x 64 example:

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Figure 5: 64 consecutive integers, the boldface entries 197–255 and 0–4, represent each of the 64 $(2 \times 2)$ squares (regular holographic sampling).
The regular (self-similar) sampling strategy is nice but can lead to Moiré's if there is some pattern in the image. Hence, idea:

**RANDOMIZE**

choose different rules at various levels of the recursive generation of the sampling order. but we want order so that we'll learn how to reconstruct the sampling sequence (i.e. we will not need to add location info to the samples!)
One possibility is:

Define:

1) \( h_{6i+p}(0) = 0 \)
2) \( h_{6i+p}(4k+m) = h_{i+m}(m) \)
3) \( g_{6i+r}(m) = h_{r+f(r)}(m) \)

\[ f(r) \frac{1}{2} \frac{3}{4} \frac{5}{6} \]

4) \( f(m_a) = \left[ g_{a+1}(m) + \sum_{b=1}^{a} h_{a-b+1} \left( \frac{m_a}{4b} \right) \right] \mod 4 \)

5) \( r(m,a) = \begin{cases} 0 & f(m_a) = 0 \text{ or } 1 \\ 1 & f(m_a) = 2 \text{ or } 3 \end{cases} \)
\( c(m,a) = \begin{cases} 0 & f(m_a) = 0 \text{ or } 1 \\ 1 & f(m_a) = 2 \text{ or } 3 \end{cases} \)

6) Then the integer \( m \) is located at \((i,j)\)
\( i = \sum_{a=0}^{N-1} 2^{N-a-1} r(m,a), \ j = \sum_{a=0}^{N} 2^{N-a-1} c(m,a) \)
Figure 1: Possible pixel ordering for a $4 \times 4$ image and the corresponding pixel sequence
Figure 9: 64 consecutive integers, the boldface entries 197–255 and 0–4, represent each of the 64 2 x 2 squares.
With this type of sampling policy we have the following **Uniformity Theorem**

Any sequence of integers of length $2^m \times 2^m$ will have exactly one representative in each of the $2^m \times 2^m$ square arrays of pixels obtained by dividing the original $2^N \times 2^N$ image evenly along the two axes. ($m < N$)
Fig. 6. (a)–(b) Original test images of size $256 \times 256$. (c)–(g) Test images of size $512 \times 512$. (a)–(d) are synthetic, with (a) and (c) having three distinct gray levels, while (b) and (d) have continuous tones. (e) is a scanned engraving. (f) is the Barbara image. (g) is the Goldhill image.
Fig. 19. Effects of quantization in the holographic representation domain. (a) Original Barbara image. (b) Image recovered with double precision. (c)–(f) Images recovered using $8 + 8$, $6 + 6$, $4 + 4$, and $2 + 2$ b/pixel, respectively. The RMSE’s of (b)–(f) are 0.003, 0.004, 0.016, 0.061, and 0.234.
Fig. 17. Effects of quantization in the holographic representation domain. (a) Original image. (b) Image recovered with double precision. (c)–(f) Images recovered using $8 + 8$, $6 + 6$, $4 + 4$, and $2 + 2$ b/pixel, respectively. The RMSE’s of (b)–(f) are 0.003, 0.005, 0.016, 0.060, and 0.231.
Fig. 18. Recovery of image from an arbitrary $128 \times 128$ portion of $H(u, v)$ using varying precision in the holographic representation. (a) Double precision. (b)–(f) $8 + 8, 6 + 6, 4 + 4, 2 + 2,$ and $1 + 1$ b/pixel, respectively. The RMSE's are 0.289, 0.289, 0.289, 0.292, 0.338, and 0.465.
Fig. 22. Reconstruction of the scanned engraving of Fig. 6(e) from arbitrary portions of \( H(u, v) \) of various sizes. (a) 256 \( \times \) 256. (b) 128 \( \times \) 256. (c) 128 \( \times \) 128. (d) 128 \( \times \) 64. (e) 64 \( \times \) 64. (f) 32 \( \times \) 64. The RMSE’s are 0.178, 0.184, 0.187, 0.190, 0.191, and 0.194, respectively.
Some experiments

- Comparing 'systematic' to 'randomized' holographic sampling
- Showing progressive refinement and insensitivity to ORDER
Fig. 11.  (a) $64 \times 64$ reconstruction using regular holographic sampling. (b) $64 \times 64$ reconstruction using "random" sampling. (c) $64 \times 64$ reconstruction using $1/d$ smoothing. (d) $64 \times 64$ reconstruction using $1/d^2$ smoothing. The RMSE's are 0.143, 0.142, 0.115, and 0.120, respectively.
Fig. 20. (a), (c), (e) Comparison of image recovery by zero padding and (b), (d), (f) by using Fourier transform directly and expanding the result by pixel replication. (a)–(b) 128 × 128 portions. (c)–(d) 64 × 64 portions. (e)–(f) 32 × 32 portions. The RMSE’s of (a)–(f) are 0.201, 0.203, 0.210, 0.224, 0.233, and 0.264.
**Insensitivity to position of rep chosen:**

Fig. 12.  (a) 128 × 128 reconstruction using 1/d smoothing and initial pixel 27,561.  (b) 128 × 128 reconstruction using 1/d smoothing and initial pixel 133,476.  (c) 128 × 128 reconstruction using 1/d smoothing and initial pixel 193,002.  (d) 128 × 128 reconstruction using 1/d smoothing and initial pixel 241,785. The RMSE’s are 0.113, 0.112, 0.112, and 0.112, which is indicative of how similar the distorted images are regardless of the starting point of the pixel sequence chosen.
Fig. 13. These images are obtained from using pixel sequences of length equal to (a) 1%, (b) 2%, (c) 3%, (d) 4%, (e) 5%, (f) 6%, (g) 7%, (h) 8%, of the number of pixels in the original 512 × 512 image. 1/d smoothing is used in all cases. The RMSE’s are 0.323, 0.248, 0.231, 0.225, 0.218, 0.213, 0.180, and 0.172, respectively.
Conclusions

- Holographic representations are a useful concept (internet applications)
- Anti-hierarchical representations should be given more attention.

[we are looking at anti-KL representations for classes of sectors.]

Ref: B. Hulka, Retravel, Holographic AncePep's, TM, Oct 85
- Ng + Karakèsct Nonredundant AngePep's, 1987
Fig. 12. (a) $128 \times 128$ reconstruction using $1/d$ smoothing and initial pixel 27561. (b) $128 \times 128$ reconstruction using $1/d$ smoothing and initial pixel 133476. (c) $128 \times 128$ reconstruction using $1/d$ smoothing and initial pixel 193002. (d) $128 \times 128$ reconstruction using $1/d$ smoothing and initial pixel 241785. The RMSE's are 0.113, 0.112, 0.112, and 0.112, which is indicative of how similar the distorted images are regardless of the starting point of the pixel sequence chosen.