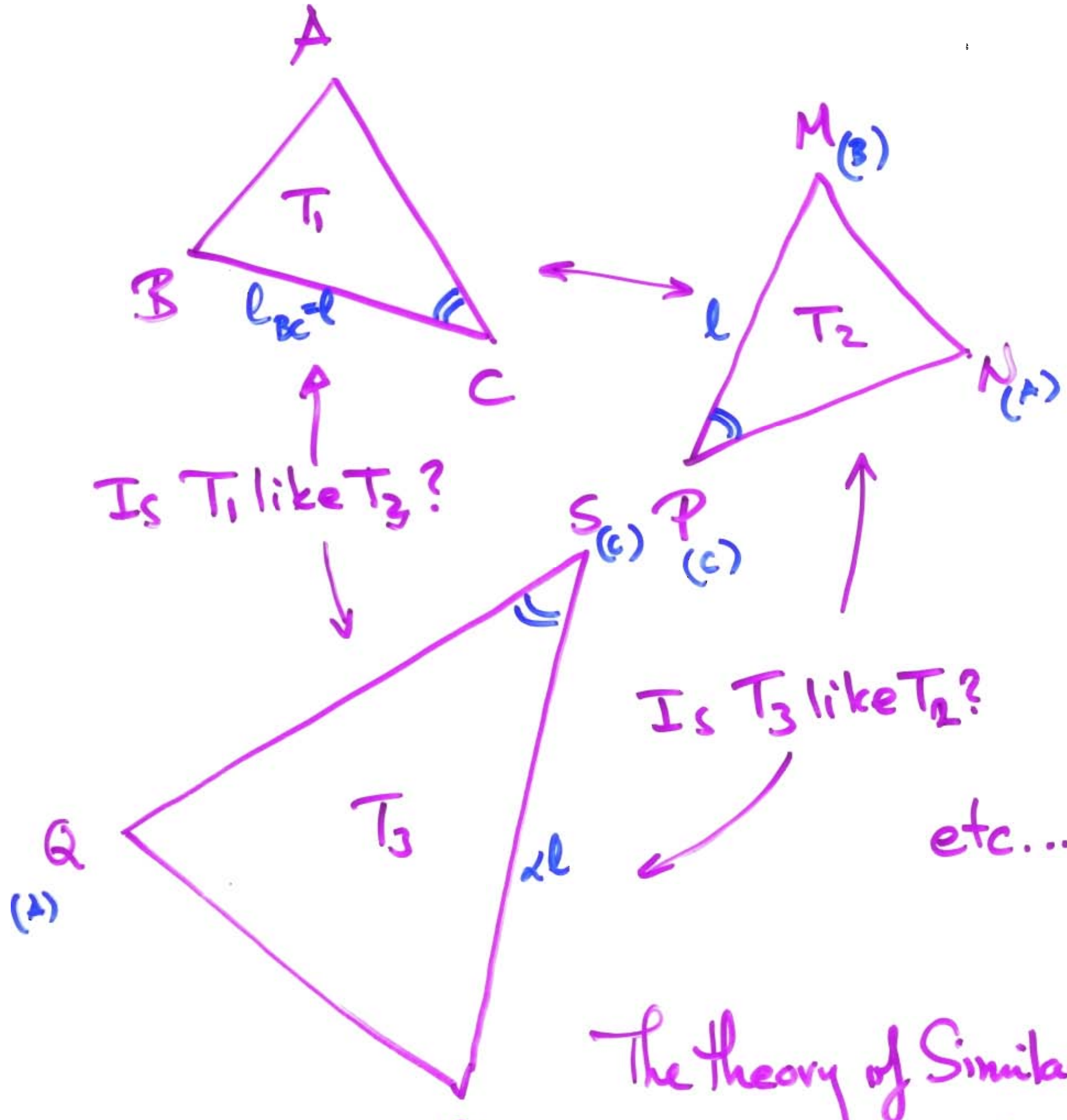
GEOMETRIC INVARIANTS & APPLICATIONS

A. BRUCKSTEIN

'High-School' Planar Geometry

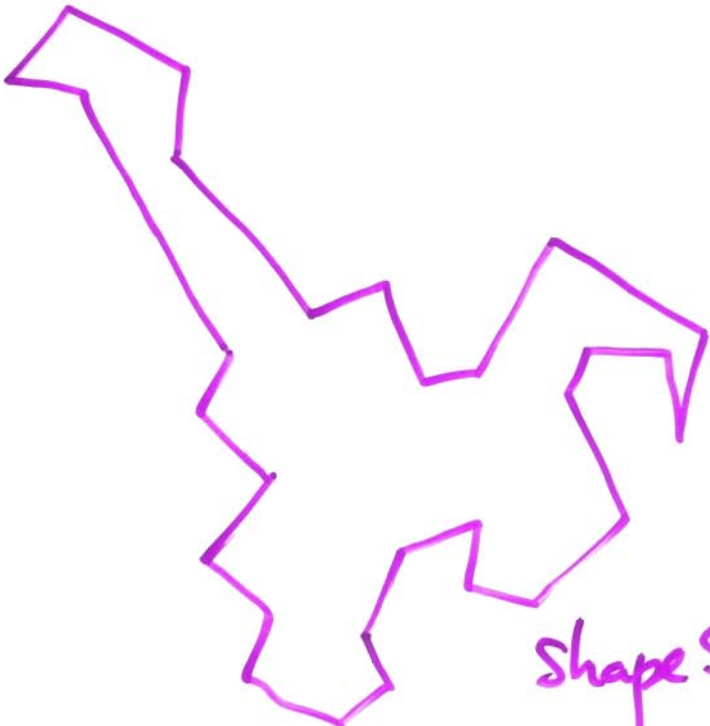


The theory of Similar Triangles.

"ACADEMIC" RESEARCH

Shape S_1

Is Shape S_2 the same as S_1?



Shape S_2

The "advanced theory" of similar Shapes, in Planar Shape Analysis – Computer Vision.

The answer to both questions:

via the theory of Geometric Invariants

Geometric Invariants

- quantities that do not change under some classes of transformations -

Mathematical Jargon:

- given a group of transformations (parametrized, continuous: Euclidean, Similarity, Affine, Projective)

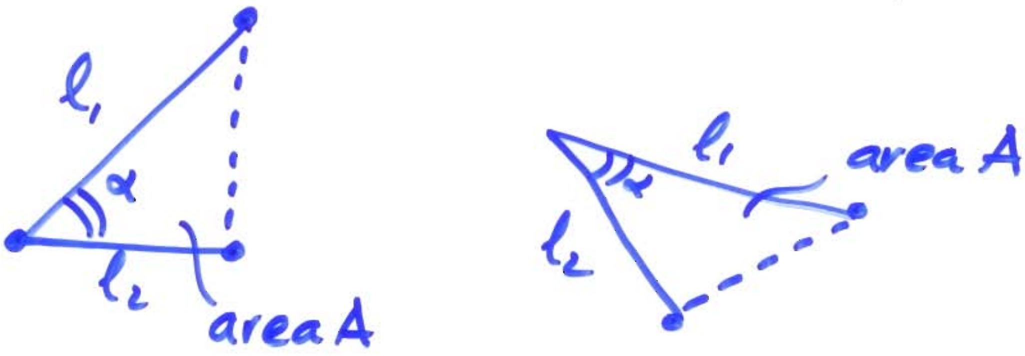
  $T_\psi : \mathbb{R}^2 \to \mathbb{R}^2$

what geometric quantities in the plane remain invariant if every point P is mapped to $T_\psi(P)$?

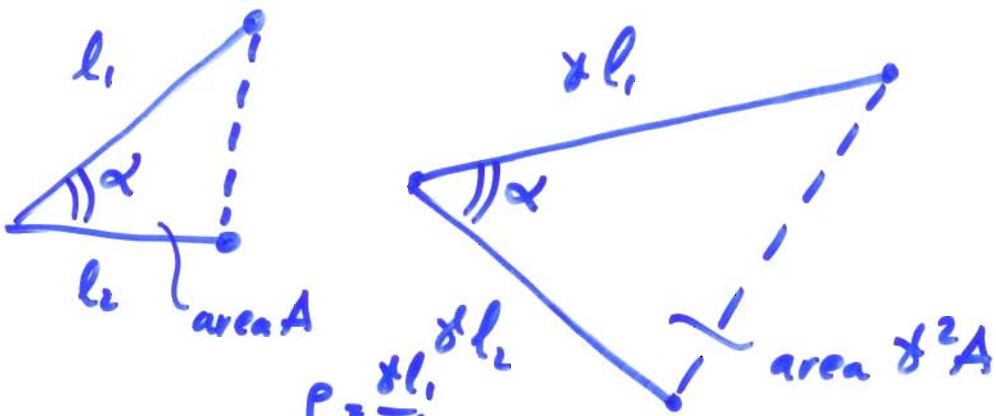
[geometry involves length, angles (metrics)
(topology deals with connectedness, holes, etc...)]

Examples of Invariants
(points & line segments).

- Euclidean Transformations



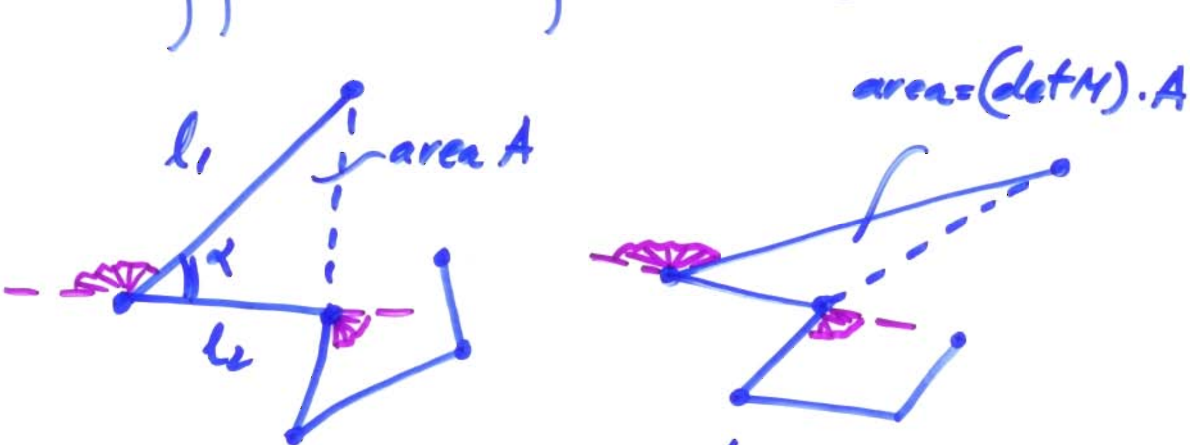
- Similarity Transformations



$\rho = \frac{l_1}{l_2}$ $\rho = \frac{\gamma l_1}{\gamma l_2}$

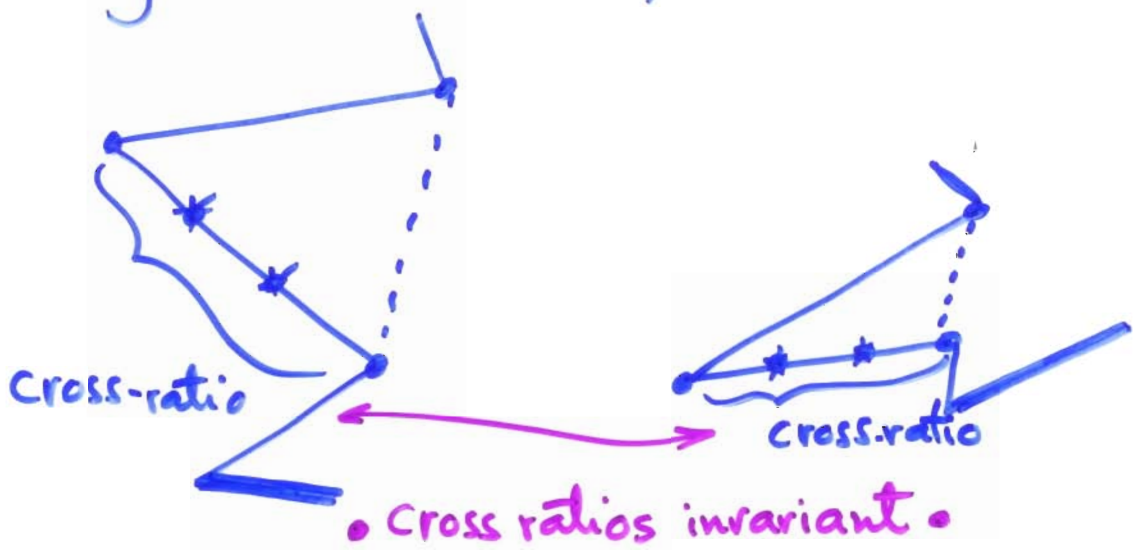
area ratios invariant.

- Affine Transformations



area ratios invariant.

• Projective Transformations



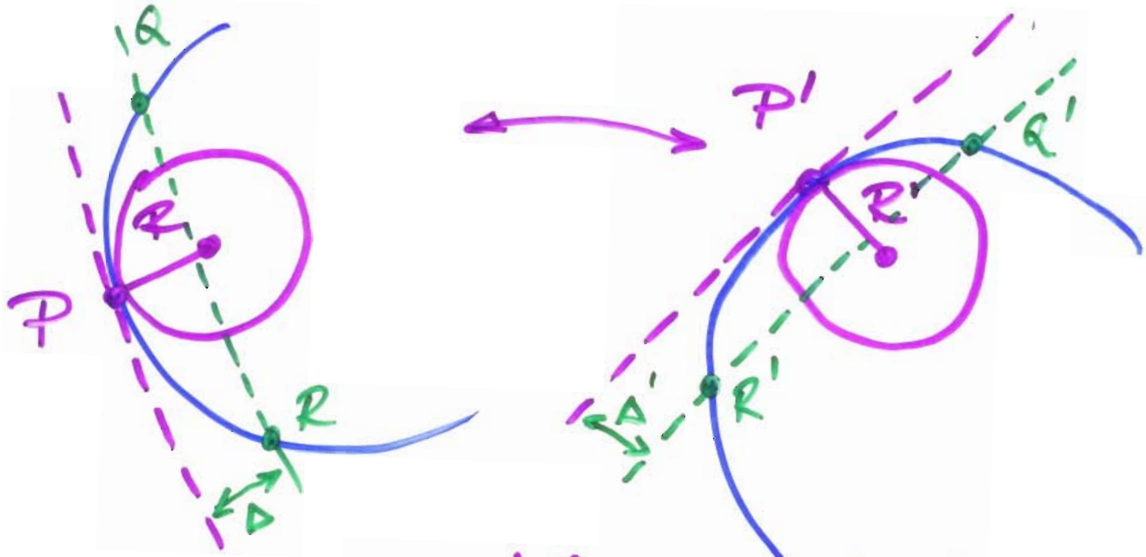
These examples show the phenomenon that more complex transformations require more sophisticated combinations of features to produce geometric invariants.

All the above for the case of points & straight lines (edges) etc...

What about curves?

for Curves

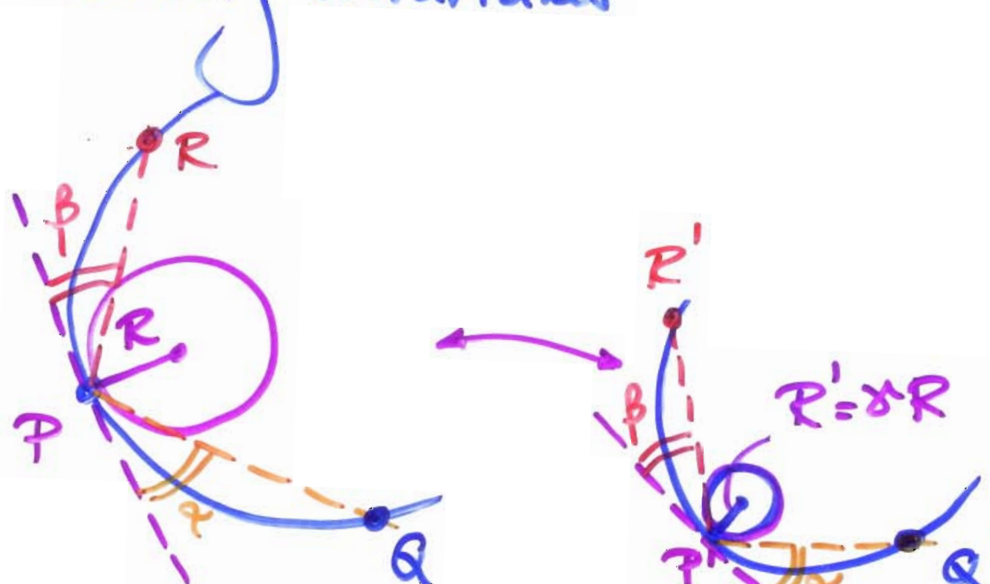
- Euclidean Invariants



$R = R'$ (equal curvature)

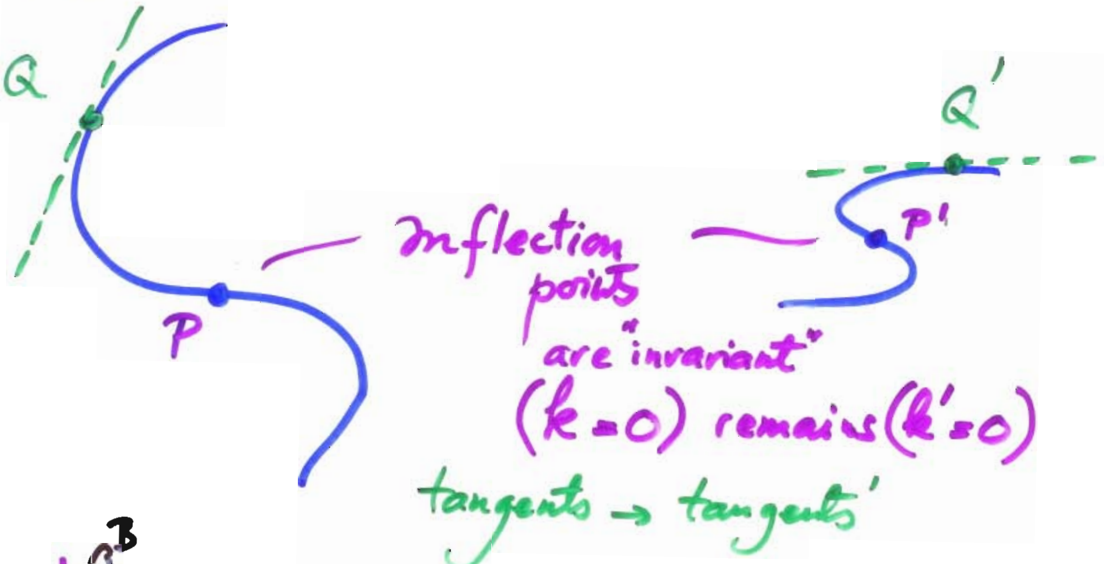
$\Delta = \Delta' \Rightarrow \blacktriangle PQR \cong \blacktriangle P'Q'R'$

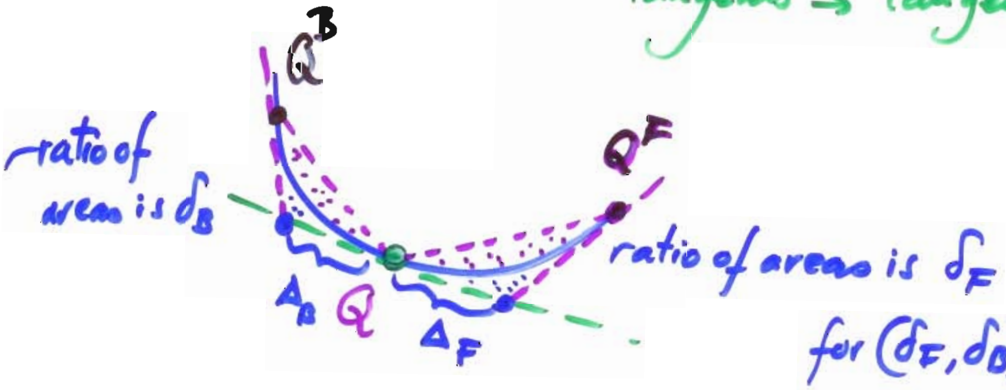
- Similarity Invariants



Now $\dfrac{l(PQ)}{l(PR)} = \dfrac{l(P'Q')}{l(P'R')}$ is an invariant.

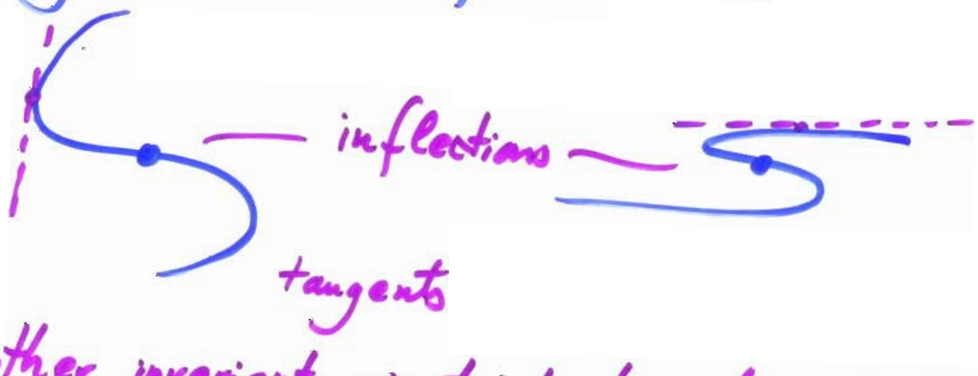
- Affine Transformations





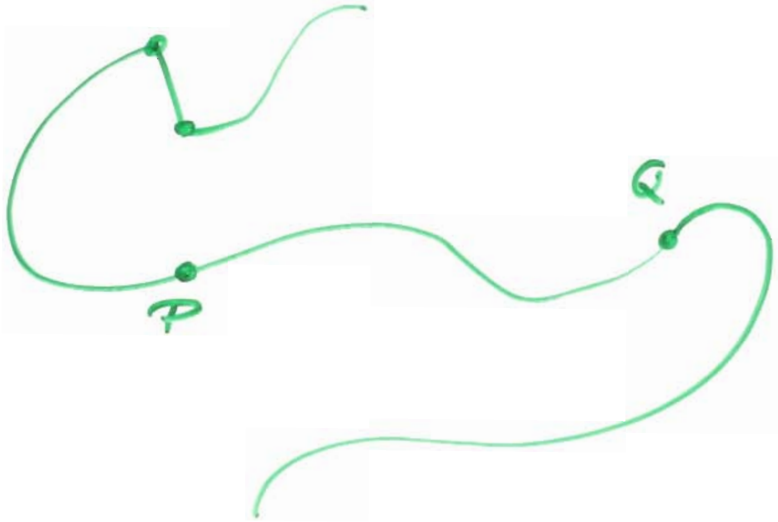
for (δ_F, δ_B) predetermined
$\frac{\Delta_F}{\Delta_B}$ <u>is</u> invariant.

- Projective Transformations



other invariants via tricks based on cross-ratios.

Curves - cont'd



continuous & differentiable portions
- can produce invariants based on "Derivatives".

Differential Geometry:

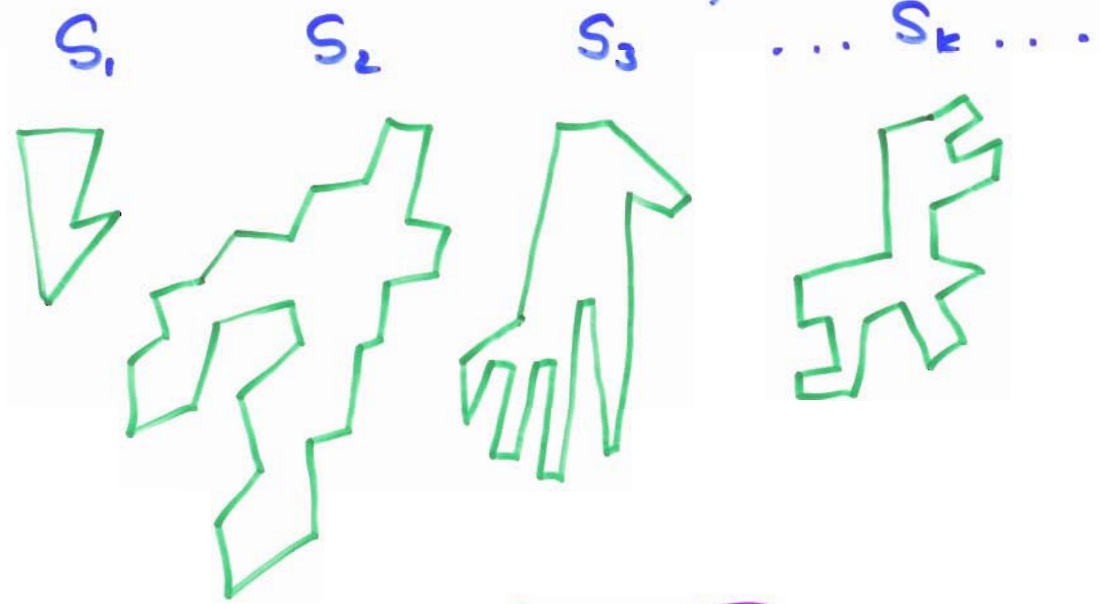
Euclidean : $k(s)$ representations (2)
Similarity : ? (3)
Affine : ? (5)
Projective : ? (7) Similar representations using horrendous #'s of derivatives

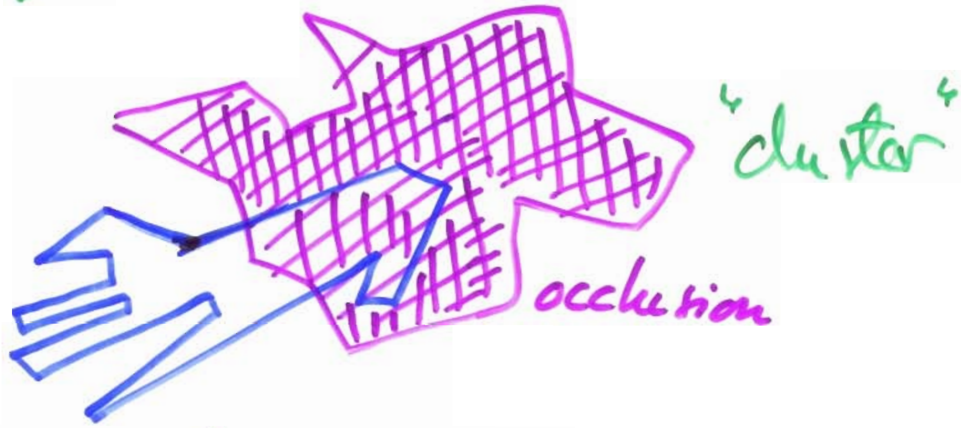
Those were studied by mathematicians to carry out the Klein programme:
"Geometry is the study of Invariants under various groups of Transformations".

APPLICATIONS

Model based object recognition under partial occlusions & viewing transformations.

Models: a series of shapes (polygonal)





How do we solve this problem?

Represent the contour of each object via a sequence of invariants under the viewing transformations

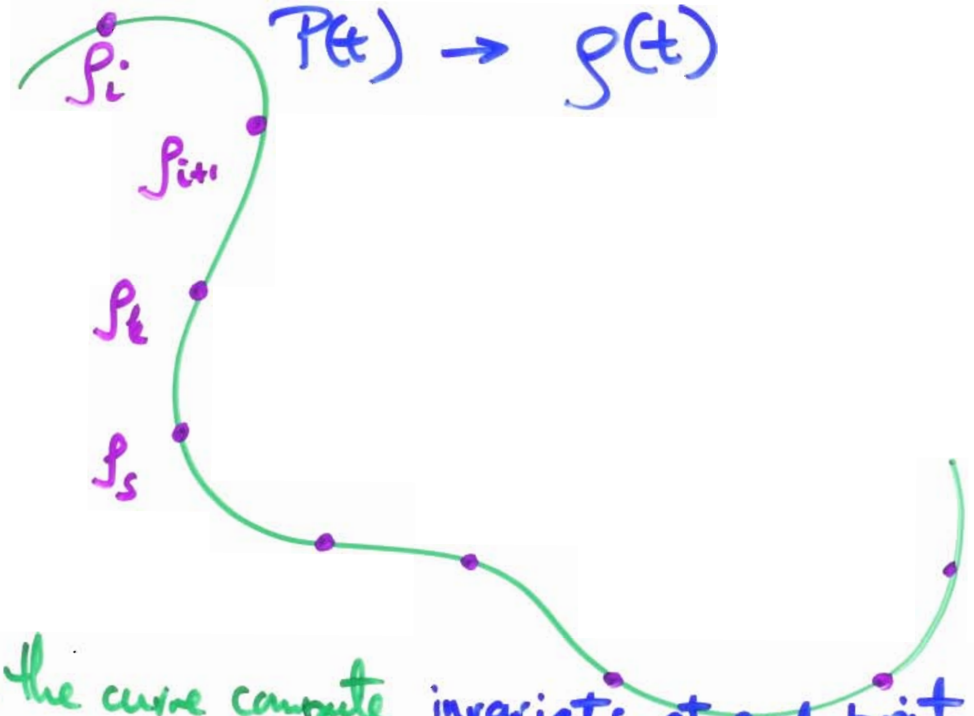
Then compute the sequence of invariants representing the cluster — A PARTIAL MATCHING PROCESS DOES THE OBJECT RECOGNITION!

If shapes are not polygonal:

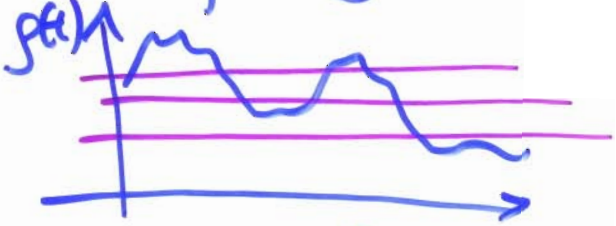
use invariant "curvature" vs "arclength" representations derived via an analysis of the differential invariants under the viewing transformations.

or:

$P(t) \to \rho(t)$



on the curve compute invariants at each point $\rho(t)$ (say the $\frac{\Delta_F}{\Delta_B}$ affine invariant).



choose levels of ρ : ρ_i

and at crossing point set labels on the curve. These points can be used as a polygonal model of objects!

$\Downarrow$ We are back to the POLYGONAL CASE.

Invariance became a "BUZZ WORD" in CV

Every day some old math results are rediscovered & applied.

(RESEARCH ≡ Re... Search for old results).

Other types of results:

- Conics remain conics under perspective
- Algebraic invariants for implicit curve representations
- Invariants for image unwarping etc...
- Invariant Curve Evolutions for Shape Analysis (Morphology).
- Invariants for 3D object recognitions

i.e. Invariants in the PUREST of CLASSICAL SENSE (GEOMETRY) are IN.