Crazy Cuts:

Dissecting Planar Shapes into Two Identical Parts

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The Problem:

Given a planar shape $S$, determine a cutting curve that divides the shape into two identical parts (up to rotations and translations; Euclidean transformation) if possible!!!
The Problem:

"The Crazy-Cuts"

Given a planar shape $S$, determine a cutting curve that divides the shape into two identical parts (up to rotations and translations; Euclidean transformation) if possible!!!
Some more crazy cut challenges
Some more crazy cut challenges and their solution cuts!
Draw the figure as shown in the illustration or just print it out.

The goal is to make a cut (or draw one line) - of course it needn't be straight - that will divide the figure into two identical parts.
**PROBLEM:** find an algorithm to determine a crazy-cut efficiently or decide that such a division is not possible.

**Prior art:**
- K. Eriksson: Splitting a Polygon into Two Congruent Pieces, ANS North, 1996
- C. Rote: Some Thought about ..., 1997
- D. El-Khetchen... et al., Partitioning a Polygon into Two Congruent pieces, CGGT, Kyoto, 2007

**Conclusion:** Efficient algorithm exists. Arguments rather complicated, long proofs...
Our Point of View: simple and cute!

If a shape has a crazy cut split into two identical pieces, this shape is a result of putting together a jigsaw-puzzle of two identical shapes!

Therefore let us first analyse the problem of Self-Docking of two Shapes (and in particular of two identical Shapes!).
SHAPES → boundary descriptors

Euclidean Invariant "SIGNATURES" of the Shape
Docking of Shapes
Docking Grammar:

Boundary of $S_I$: $\ldots \mathcal{P}^I J \ldots$

Boundary of $S_{II}$: $\ldots \mathcal{P}^I \overline{J} \ldots$

$J$ and $\overline{J}$ are characterized by

$$k^I(s) = -k^I(\Sigma - s)$$

$$\delta \in [\delta^A, \delta^B] \quad \Sigma - \delta^I_A = \delta^B_B$$

$$\Sigma - \delta^I_B = \delta^A_A$$

Boundary of Docked Shape

$$S = S_I \circlearrowleft S_{II} : \ldots \mathcal{P}^I \mathcal{P}^I \ldots$$
Self-Docking of Shapes

Docking of Shapes but $S_I = S_F$

Crucial Consequence

The dichotomy for $I$:

Either this

$\delta = 0$
$\delta = \Sigma - \delta$
$\delta = \Sigma$

Or this

$\delta = \Sigma / 2$
$\Sigma$

$k(\delta) = -k(\Sigma - \delta)$ for $\delta \in [0, \Delta]$

depending on whether $\Delta > \Sigma / 2$
JOINT SHAPE

BOUNDARY GRAMMAR

...PP...

...P|Q|Q|J|P...
SELF DOCKING THEOREM:

A PLANAR SHAPE

EITHER DOCKS TO ITSELF OVER
DISJOINT (J's) MATCHING PORTIONS
OF ITS BOUNDARY : \( J \) and \( \bar{J} \)

OR DOCKS TO ITSELF OVER
THE SAME "SELF-MATCHING" PORTION
OF ITS BOUNDARY : \( J = \bar{J} \)

- NO MIXED SELF DOCKING
  BOUNDARY PORTIONS.
Solution for Crazy Cut

For the given shape

- find the boundary signature string \( k(s) \) (periodic with period \( L \) (length))
- detect whether \( k(s) \) has the form \( \ldots PP \ldots \) or \( \ldots PJQQJP \ldots \) with an \( O(L^3) \) algorithm!
- if \( PP \) : all J's are OK that do not self intersect
  if \( PJQQJP \) : cut is J and \( \ldots PJQJ \ldots \) is the piece
  (Test for validity fast).
Conclusions

- Crazy cut solutions easy from Self Docking analysis

- Crucial role played by BOUNDARY SIGNATURES like in our previous work on (skew) Symmetry detection

- One can solve crazy cuts for shapes distorted by viewing transformations too!

via IN Variant SIGNATURES

SHAPE ANALYSIS = STRING PROCESSING