CRAZY CUTS: DISSECTING PLANAR SHAPES INTO TWO IDENTICAL PARTS

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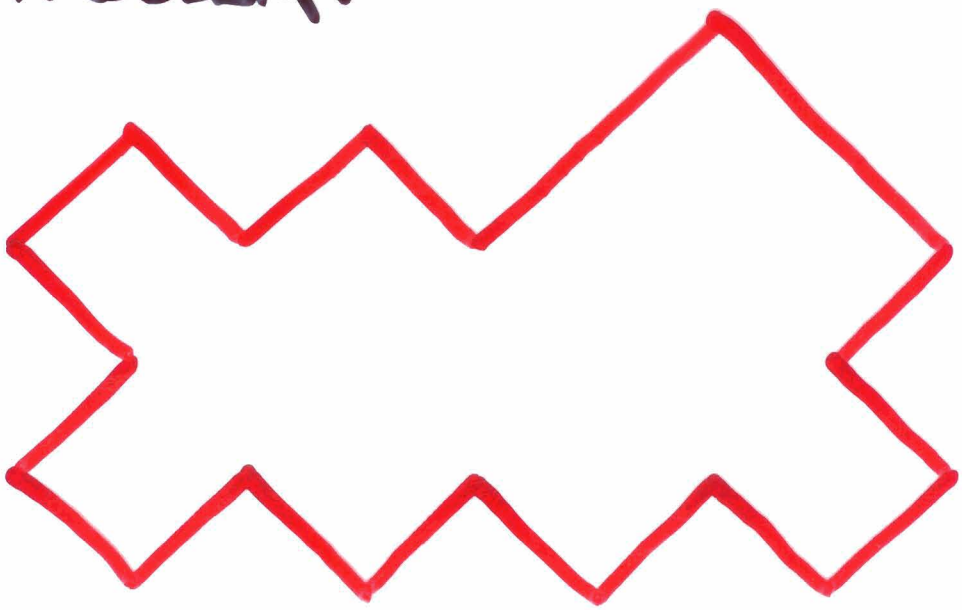
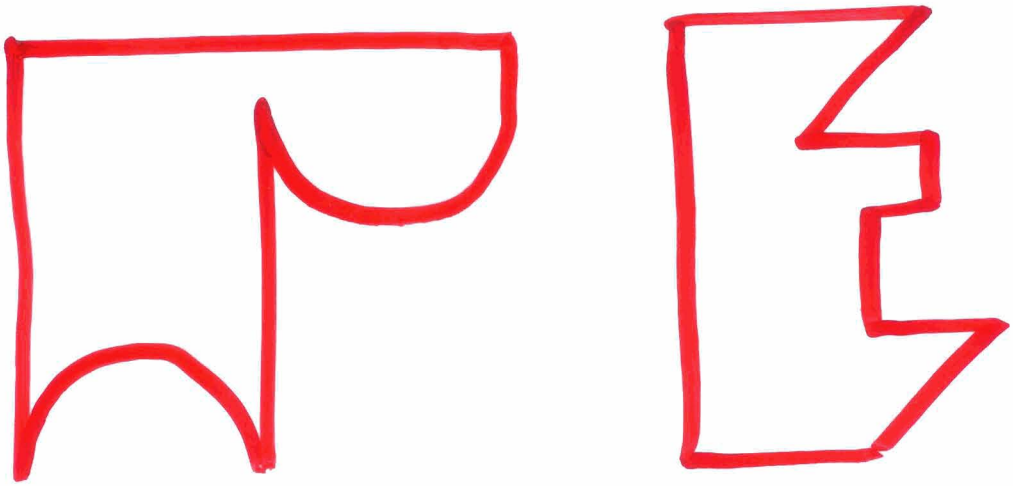
THE PROBLEM:





GIVEN A PLANAR SHAPE S, DETERMINE A CUTTING CURVE THAT DIVIDES THE SHAPE INTO TWO IDENTICAL PARTS (up to rotations & translations; Euclidean transf.) if possible!!!

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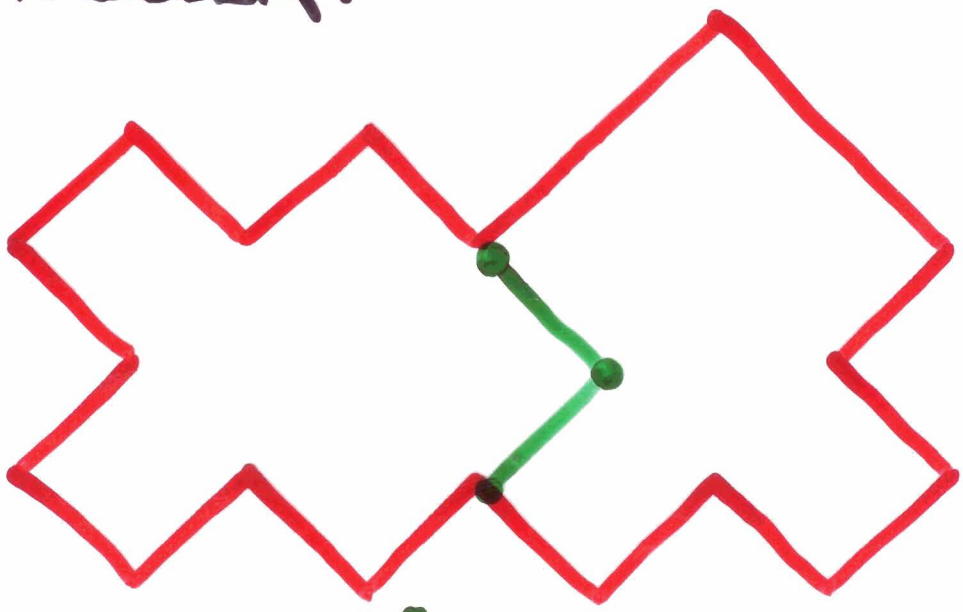
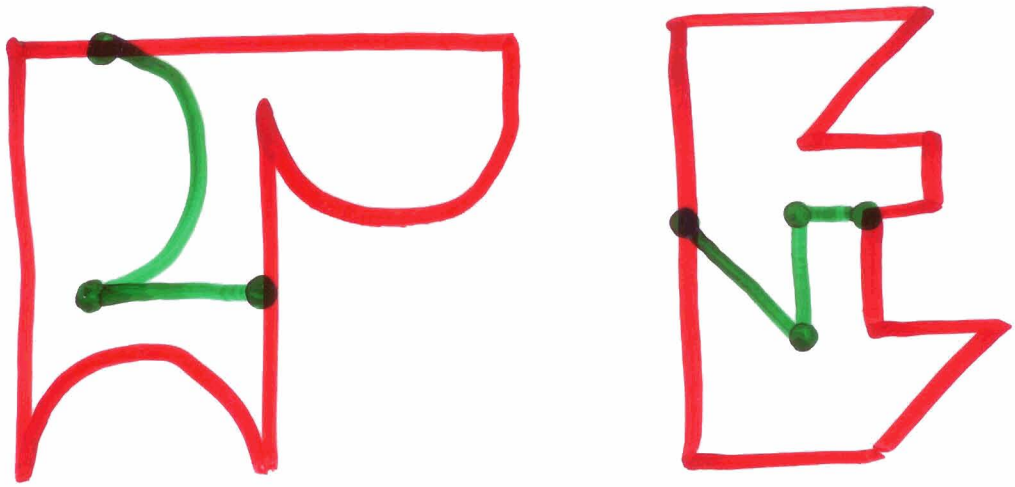


"THE CRAZY-CUTS"



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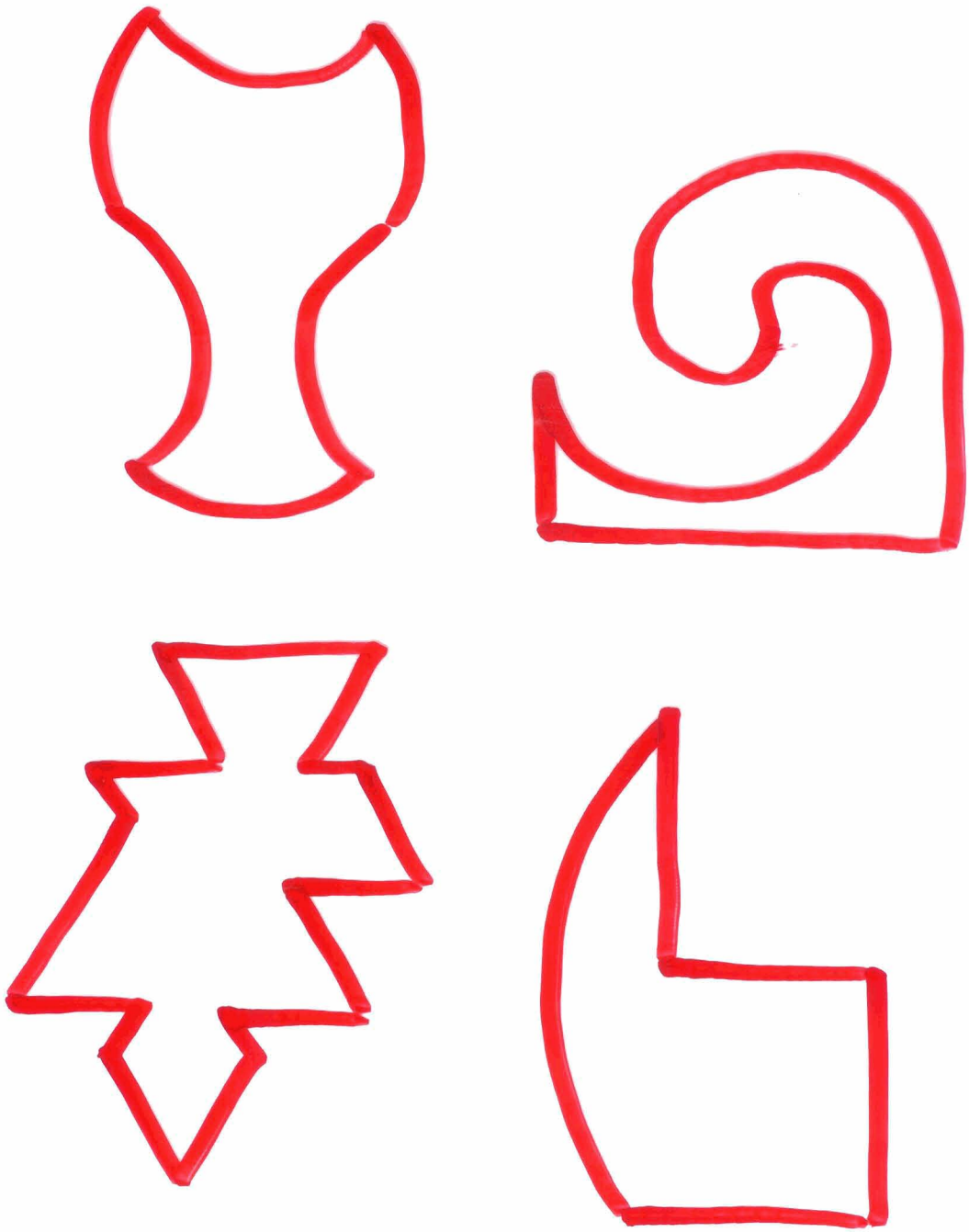
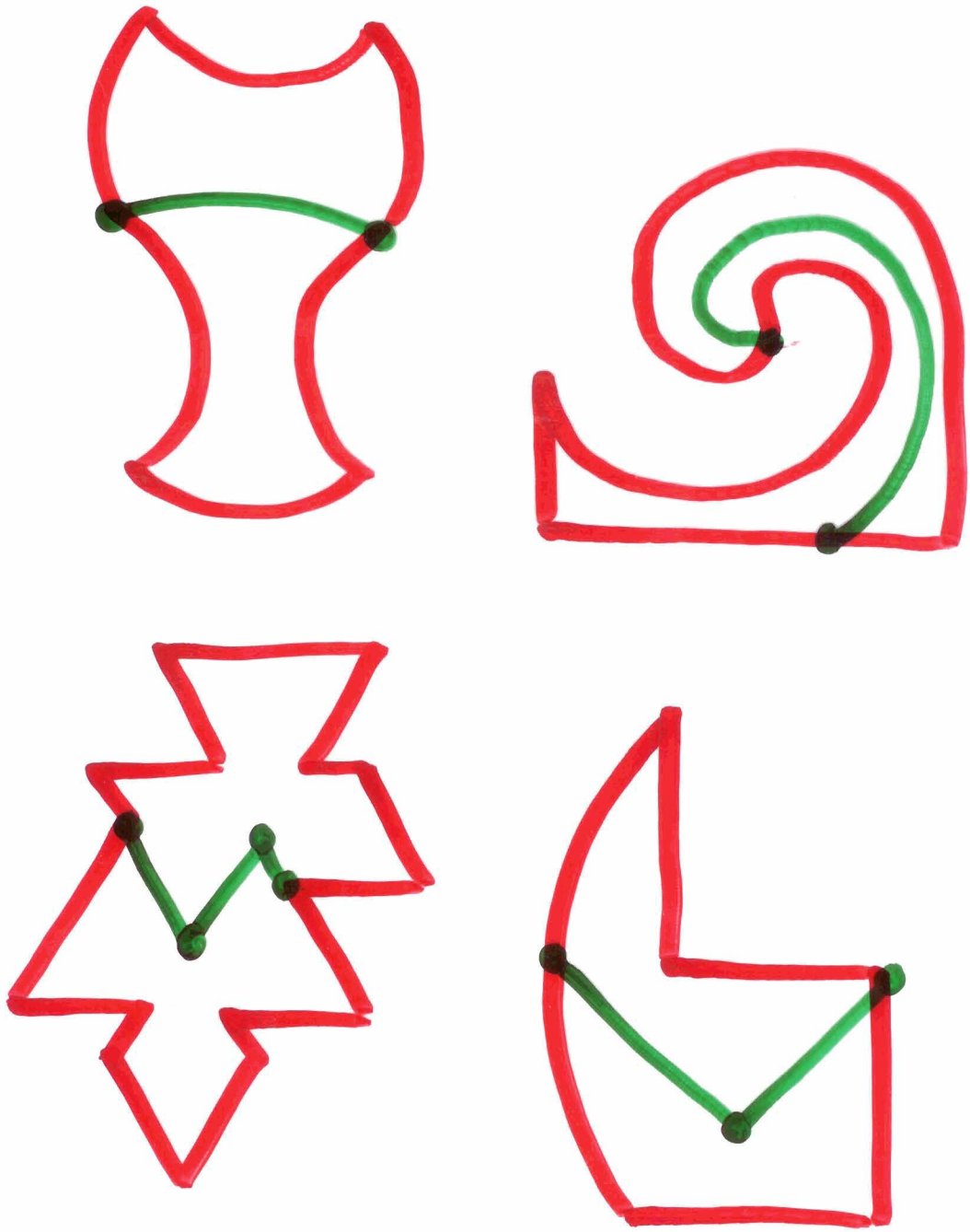


SOME MORE CRAZY CUT CHALLENGES



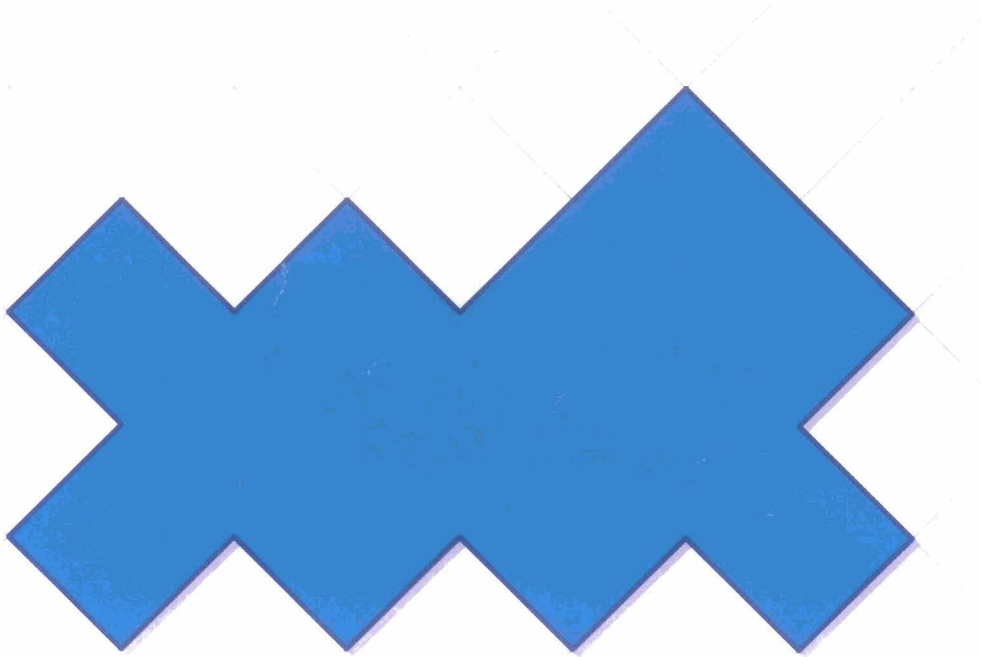
SOME MORE CRAZY CUT CHALLENGES

and their solution cuts!

Crazy Cut

after Martin Gardner

Puzzles.COM



Draw the figure as shown in the illustration or just print it out.

The goal is to make a cut (or draw one line) - of course it needn't be straight - that will divide the figure into two identical parts.

Last Updated: August 11, 2005

PROBLEM: find an algorithm to determine a crazy-cut efficiently or decide that such a division is not possible.

Prior art:

- K. Eriksson: Splitting a Polygon into Two Congruent Pieces, AMMonthly 1996
- G. Rote: Some thought about ... 1997
- D. El-Khetchen ... et al, Partitioning a Polygon into Two Congruent pieces CGGT, Kyoto, 2007

Conclusion: Efficient Algorithm Exists

Arguments Rather Complicated, Long Proofs ...

Our Point of View:

simple and cute!

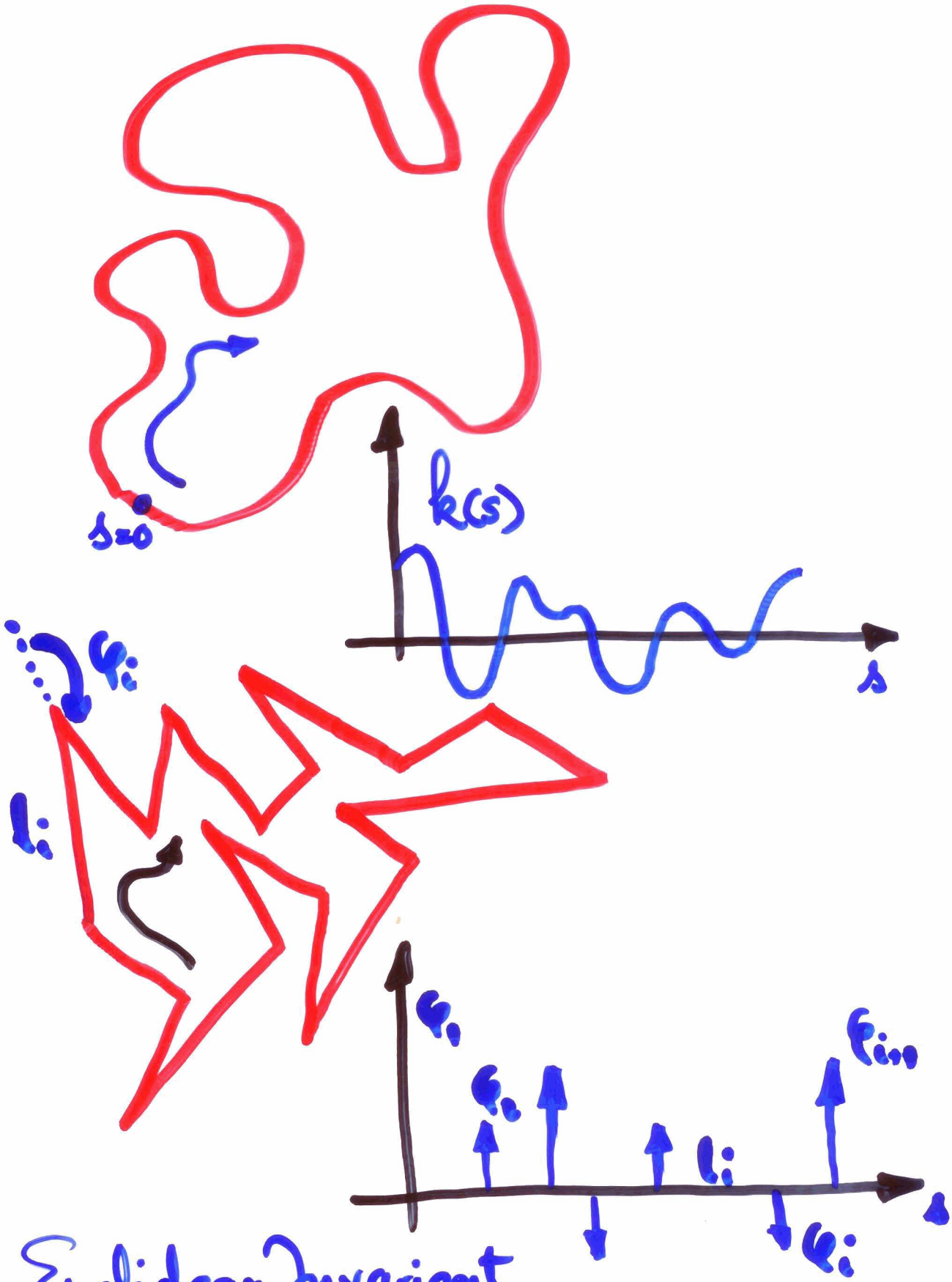
If a shape has a crazy cut split into TWO identical pieces, this shape is a result of putting together a jigsaw-puzzle of two identical shapes!

Therefore let us first analyse the problem of

Self-Docking of two Shapes

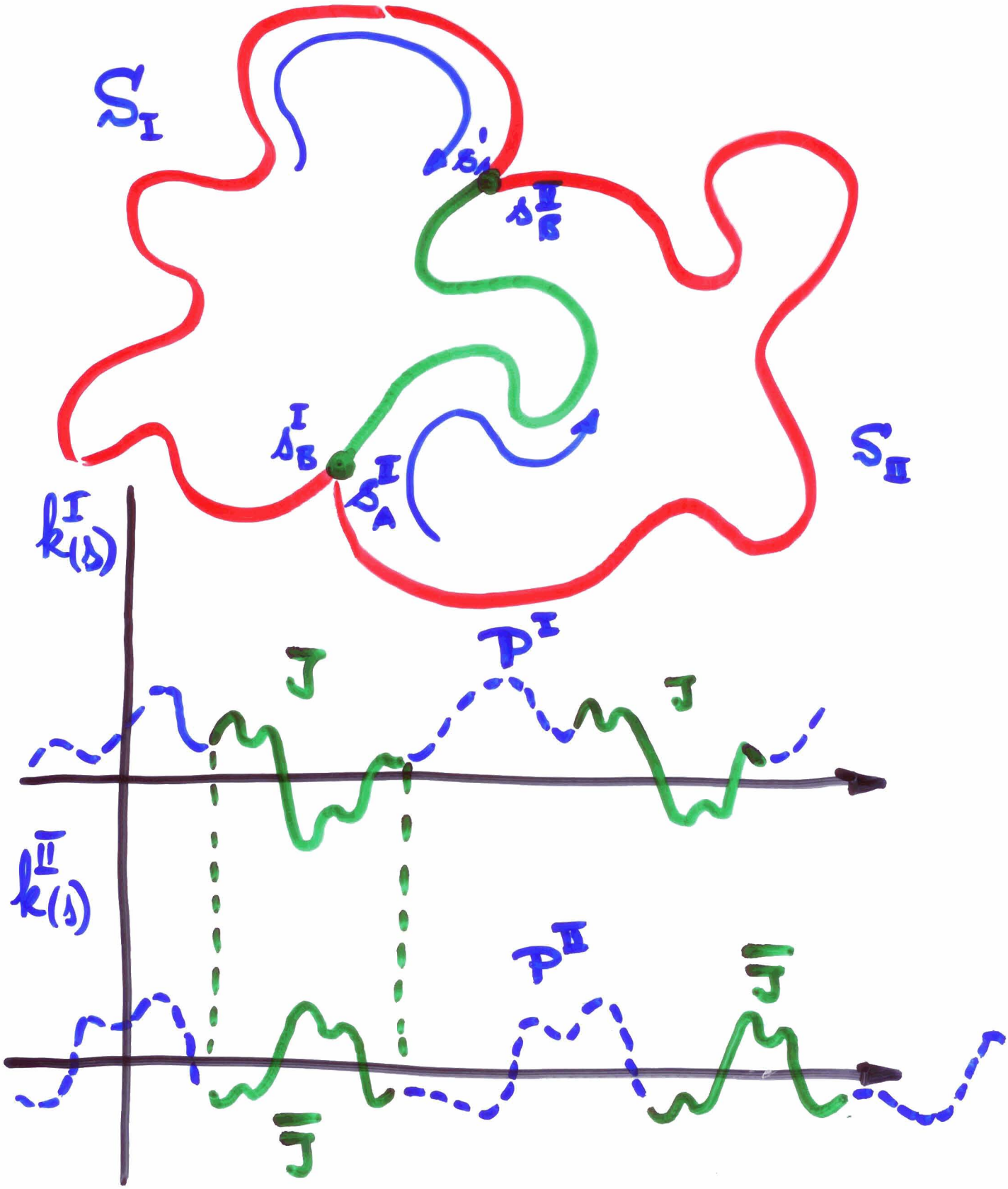
(and in particular of two Identical Shapes!).

SHAPES → boundary descriptors



Euclidean Invariant "SIGNATURES" of the Shape

DOCKING OF SHAPES



DOCKING GRAMMAR:

Boundary of S_I : $\ldots P^I J \ldots$

Boundary of S_{II} : $\ldots P^{II} \bar{J} \ldots$

J and $\bar{J}$ are characterized by

$$k^I(s) = -k^{II}(\Sigma - s)$$

$$s \in [s_A^I, s_B^I] \qquad \Sigma - s_A^I = s_B^{II}$$

$$\Sigma - s_B^I = s_A^{II}$$

Boundary of Docked Shape

$$S = S_I \, J \, S_{II} :$$

$$\ldots P^I P^{II} \ldots$$

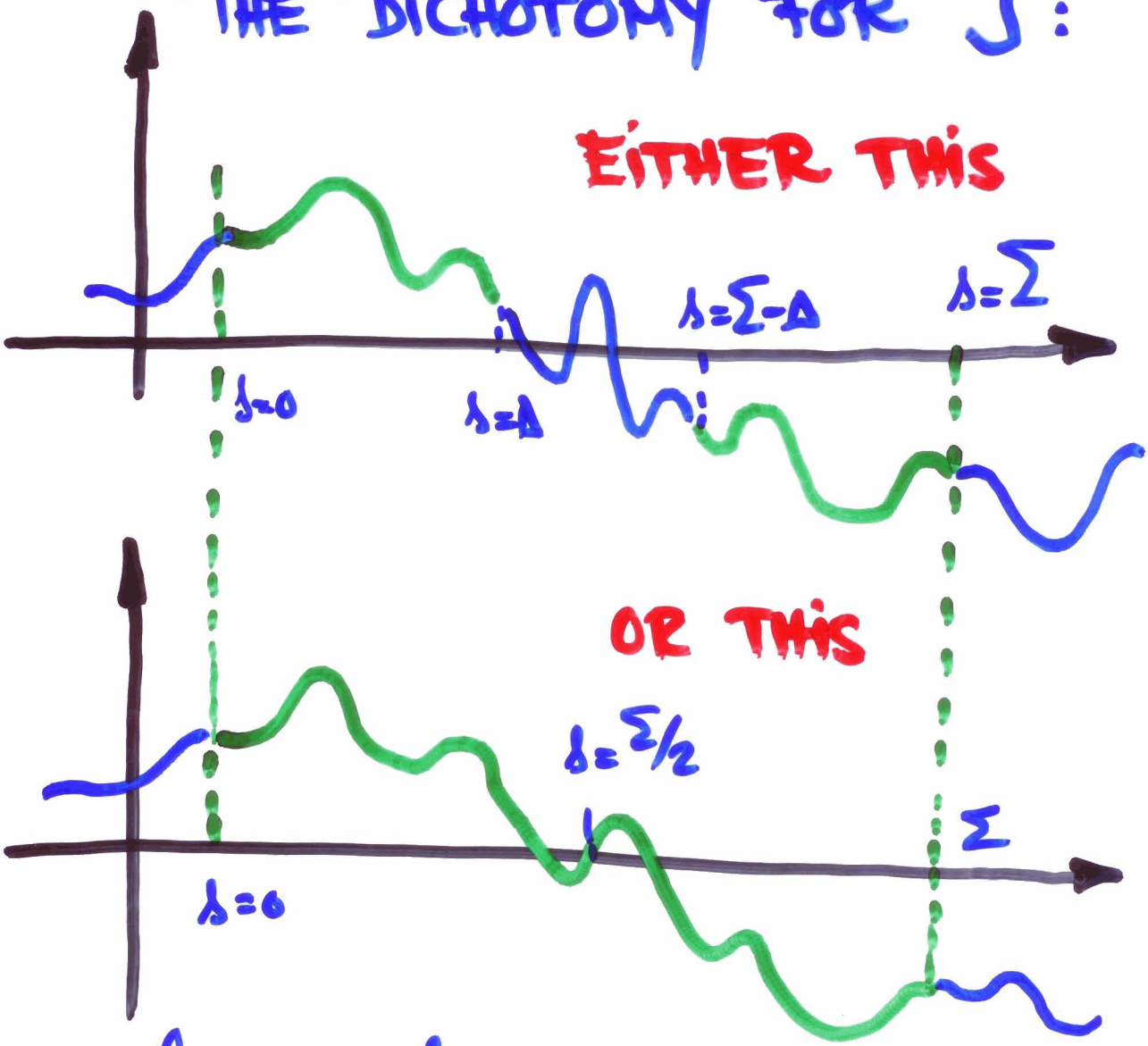
SELF-DOCKING OF SHAPES

DOCKING OF SHAPES BUT

$$S_I \equiv S_{II}$$

CRUCIAL CONSEQUENCE

THE DICHOTOMY FOR J:



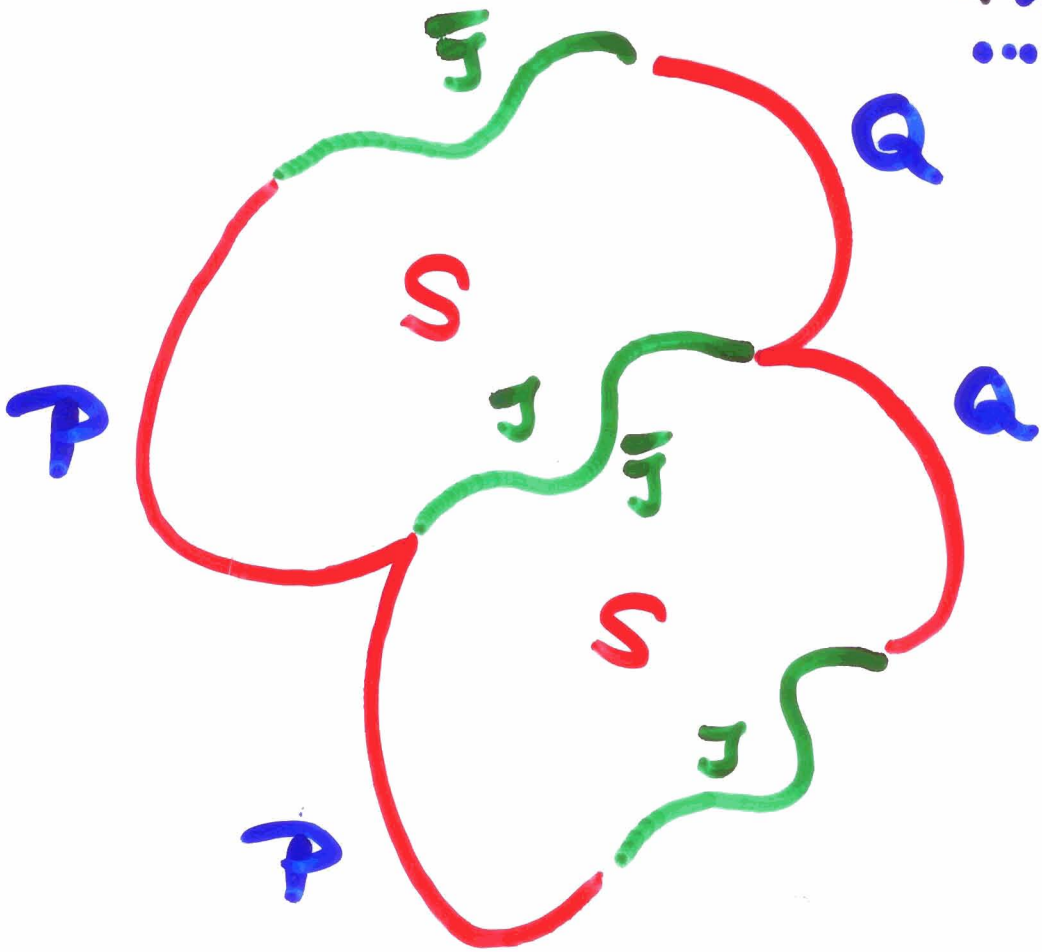
$k(s) = -k(\Sigma - s)$ for $s \in [0, \Delta]$

depending on whether $\Delta \geq \Sigma/2$

JOINT SHAPE BOUNDARY GRAMMAR

...PP....

...PJ̄QQJP...



SELF DOCKING THEOREM:

A PLANAR SHAPE

EITHER DOCKS TO ITSELF OVER DISJOINT ($\bar{J}$'S) MATCHING PORTIONS OF ITS BOUNDARY : J and $\bar{J}$

OR DOCKS TO ITSELF OVER THE SAME "SELF-MATCHING" PORTION OF ITS BOUNDARY $J \equiv \bar{J}$

- NO MIXED SELF DOCKING BOUNDARY PORTIONS.

SOLUTION FOR CRAZY CUT

For the given shape

- find the boundary signature string $k(s)$ (periodic with period L (length))
- detect whether

  $k(s)$ has the form ...PP...

  or ...P$\bar{J}$QQJP...

  with an $O(L^3)$ at most algorithm!
- if PP : all J's are OK that do not self intersect

  if P$\bar{J}$QQJP : cut is J and ...P$\bar{J}$QJ... is the puzzle PIECE

(TEST for VALIDITY fast).

CONCLUSIONS

- Crazy cut solutions easy from SELF DOCKING analysis
- Crucial role played by BOUNDARY SIGNATURES like in our previous work on (skew) Symmetry detection
- one can solve crazy cuts for Shapes distorted by viewing transformations too!

via INVARIANT SIGNATURES

SHAPE ANALYSIS
$\approx$ STRING PROCESSING