A Book on Digital Geometry

(that should be written)

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This is a talk describing a
NONEXISTENT BOOK
that should be written.

INSPIRATION:

1) Stanislaw Lem
A Perfect Vacuum (1971)

2) Enrique Vila-Matas
Bartleby & Co (2000)

"Literature to date has told us of fictitious characters. We shall go further: we shall depict fictitious books. Here is a chance to regain creative liberty, and at the same time to wed two opposing spirits – that of the belletrist and that of the critic."

"The writer loses his freedom in his own book, the critic in another's"

The book of Stanislaw Lem reviews & criticizes 16 fictitious books including his own "Perfect Vacuum" which brakes the pattern, because his book does exist!

"The glory or the merit of certain men consists in writing well; that of others consists in not writing"

Jean de la Bruyère

is the motto of Vila-Matas' book which analyses:

"... the negative impulse or attraction towards nothingness that means that certain creators, while posessing a very demanding litterary conscience (or precisely because of this), never manage to write..."

"... Only from the negative impulse, from the labyrinth of the NO, can the writing of the future appear. But what will this literature be like? ... a colleague, somewhat maliciously put this question to me.

"I DON'T KNOW" I said. "IF I KNEW I'D WRITE IT MYSELF"

Vila-Matas, Bartleby & Co (2000)

After this literary INTRODUCTION What is the ideal(?) book on

DIGITAL GEOMETRY?

What is digital geometry?

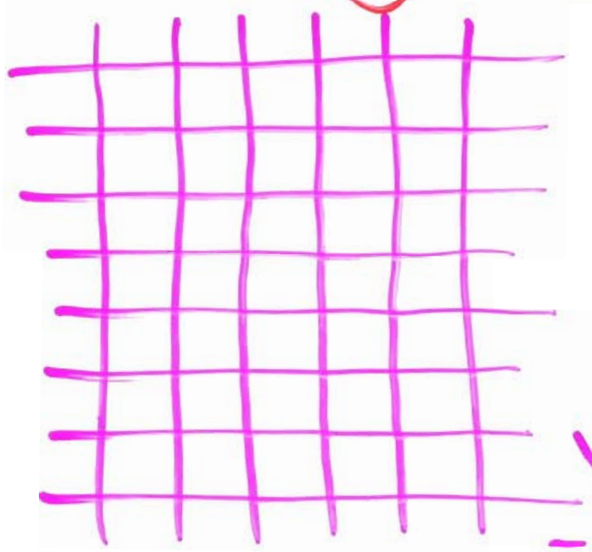
If I had an answer, I'd sit down and write the book. But let's try to see what it could be.

DIGITAL GEOMETRY

<u>what it is?</u>

in IMAGE PROCESSING / GRAPHICS / IMAGE ANALYSIS

we work on <u>grids of pixels</u>



a lot of work went into "developing" a way "to live" on these grids: to measure objects that have been "discretized", to analyse and recognize them, etc. etc. etc..

This activity produced several books:

- K. Voss : Images on $\mathbb{Z}^n$ (??)

- R. Klette & A. Rosenfeld :
  Digital Geometry for Image Analysis (??).

summarizing thousands of papers.

These are very nice books indeed, and contain a lot of information, but they are not "<u>MY BOOK</u>".

I want a book in the spirit of Hilbert:

"GEOMETRY, LIKE ARITHMETIC, REQUIRES ONLY A FEW AND SIMPLE PRINCIPLES FOR ITS LOGICAL DEVELOPMENT. THESE PRINCIPLES ARE CALLED THE <u>AXIOMS</u> OF GEOMETRY. THE ESTABLISHMENT OF THE AXIOMS OF GEOMETRY AND THE INVESTIGATION OF THEIR RELATIONSHIPS IS A PROBLEM THAT HAS BEEN TREATED IN MANY EXCELLENT WORKS ... SINCE ...EUCLID. THIS PROBLEM IS EQUIVALENT TO THE LOGICAL ANALYSIS OF OUR PERCEPTION OF SPACE".

So Let's try an outline of a basic book on Grid Geometry:

$*$1) The $\mathbb{Z}^2$ grid:

<u>POINTS</u> are (i,j) pairs $\begin{matrix} i \in \mathbb{Z} \\ j \in \mathbb{Z} \end{matrix}$

(Here we can go into the $\mathbb{Z}^2$ grid and how it is generate by the pair $(1,0)$, $(0,1)$; and how other bases are possible,

i.e. the group of $|\det| = 1$ 2×2 integer matrices $SL(2,\mathbb{Z})$.)

And now we are Stuck!

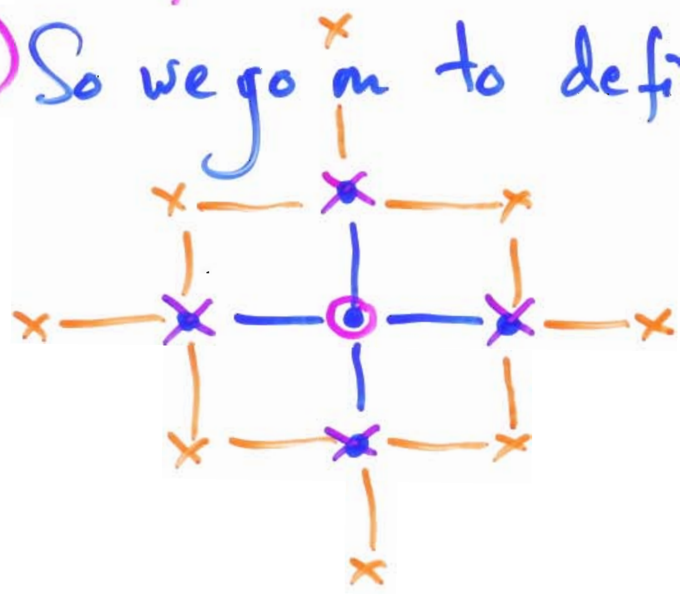
Because we have to define LINES ?

DO WE HAVE TO ??

~ do we have to follow the pattern of classical plane geometry?

~ what would our classical geometry be like if we lived on a $\mathbb{Z}^2$ grid?

*2) So we go on to define neighborhoods.



"jump" to neighbor is one step. (distance).

Using neighborhoods, and the "natural" Manhattan metric (of # of jumps) we can define

connectedness and grid shapes

polyminoes.

× 3) Connected shapes on $\mathbb{Z}^2$.

Their area (# of points

~relation to classical area Pick's Theorems)

Their perimeter

"A" a shape - (connected)

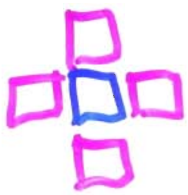
$|N(A)|$ # of grid points that are neighbors of points of A. $(\triangleq N(A))$

#4) Special Shapes

"Most condensed" shapes

Lots of shapes with area (# points) = A.

What is (are) the shapes with minimal $|N(A)|$.



$A=1$

$N(1)=4$

$A=2$

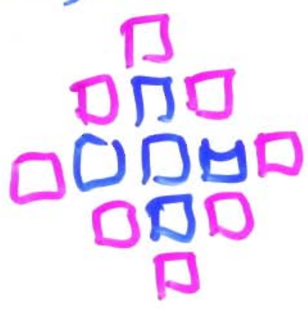
$N(2)=6$

$A=3$

$N(3)=7$

$A=4$

$N(4)=8$



$A=5$

$N(5)=8$

An interesting topic:

DISCRETE ISOPERIMETRIC PROBLEM

(was addressed in old papers, partially!)

*5) Shape Evolutions

Dilation → add to A the points that form N(A), analyse rates of grows!

Erosion → delete from A the points that are neighbors of N(A). analyse rate & patterns of disappearance.

* 6) What about LINES
do we have those?

(We have linear separability
with beautiful results!)

* 7) Duality
"Hough Space"

* 8) Shape Interactions
"Distances between shapes"

* 9) Shape similarities (?)
- rotations ($\frac{\pi}{2}$)
- flips
- translations.

OTHER TOPICS

- what the basic axioms should be?
- is the theory developable along different lines (DOES THERE EXIST A "NONEUCLIDEAN" DIGITAL GEOMETRY?)

Now you see why the Book is not <u>written</u>!