Ominview Cameras with Curved Surface Mirrors

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Abstract

Omnidirectional cameras are important for a variety of applications like videoconferencing and robot navigation. This paper discusses several ideas for building such cameras using mirrors with curved surfaces. It turns out that hyperbolic and parabolic mirror profiles are optimally suited for omniviewing. [1]

1. Introduction

Escher's work called "Hand with Reflecting Globe" (Figure 1) illustrates how a curved mirror enables us to (simultaneously) see more of the surroundings at the expense of geometric distortions. In a recent report, Vic Nalwa described the design of "the first truly omnidirectional" camera, involving planar mirrors and multiple cameras [Na96]. However, it turns out that a similar result can be achieved with a parabolic mirror and a camera that implements orthographic projection, or with a hyperbolic mirror and a camera that implements "the usual" perspective projection.

In order to have a "true omnidirectional view" one must design a device that looks around simultaneously from a single well defined viewpoint. Nalwa's paper [Na96] nicely surveys the history of omniviewing and describes the various possibilities, stressing the difficulties involved in getting "true", *single* viewpoint omnidirectionality. His design, although it requires four cameras, is clever and practical. However, one might wonder whether it would be possible to obtain the same effect with a curved mirror reflecting rays toward a single camera. Such a design would have some definite advantages: Only one camera would be required, and the image would be seamless. This paper shows that, indeed, it is possible to achieve true omnidirectional viewing with a mirror that has paraboloidal surface, by looking at if from far away, or with a device that implements orthographic projection. Furthermore, the same result can be achieved with a hyperboloidal mirror viewed via a pinhole camera that implements perspective projection. The calculations showing these results are simple and based on elementary geometric optics.

2. Omniviewing with Spheres and Cones

Suppose that a pinhole camera looks at a spherical mirror (like in Escher's drawing), as depicted in Figure 2. The image generated by the camera $I(\theta, r)$, most naturally represented in polar coordinates, will correspond to scanning the world with rays that do *NOT* originate at a well defined point. Consider next a conic mirror viewed from the apex, as was done in [YKT94], see Figure 3. This design too does not simulate looking around from a single point of view, but rather from a circular locus of effective projection centers, as pointed out in [Na96]. In this case, if the conic mirror has a slope of $\psi = 45°$ (as seen in Figure 3) we have an image $I(\theta, r)$ with $I(\theta_0, r)$ being the "central vertical slice" of the world image as would be seen with a pinhole camera having focus at a point F' located at $(-\theta_0, H)$ – in

[1]This Technical Memo was published at Bell Laboratories following a lecture by Vic Nalwa on his "true omni-directional" viewer. A patent application was filed in parallel, and in time the authors found out that the hyperbolic mirror configuration was already in use by a Japanese team led by S. Tsuji. The original part of this disclosure was eventually granted the US Patent 5,920,376, and comprises the two-parabolic-mirror-configuration and the proposal to build adapted C-MOS image sensors. In parallel to this work Shree Nayar's cameras appeared and they led to a wealth of new and exciting developments in this area. This paper should be seen as a historical document mostly, and the references in it reflect what we were aware of at the time of its writing.

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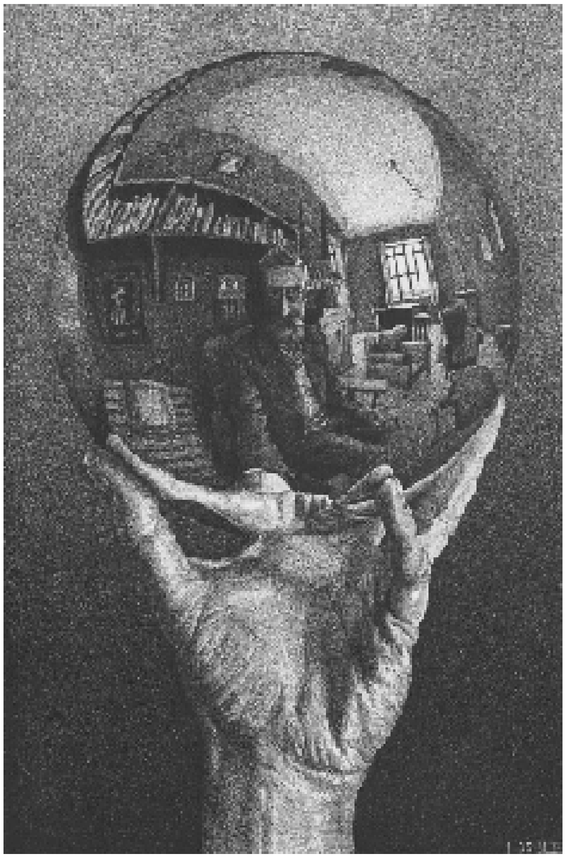

Figure 1. Escher on "Omniviewing": Hand with Reflecting Globe (A Scanned Esher Engraving)

polar coordinates – in the horizontal plane of the conic mirror's apex.

In order to get an image $I(\theta, r)$ that would virtually combine "central vertical slices" from a pinhole camera that "rotates" around a focal point F' fixed in space (on the axis of symmetry of the viewing mirror) we need to design a mirror so that the rays reflected by it originating at F (the viewing camera focal point) will all meet, when back projected, at a virtual focal point F'. The design of such a mirror profile is the main theme of this paper.

3. Parabolic and Hyperbolic Profiles

3.1. Omniviewing with Parabolic Mirrors

Our first idea for the design of a true omniviewer arises from the observation that a set of parallel rays is focused by a parabolic mirror, as seen in Figure 4.

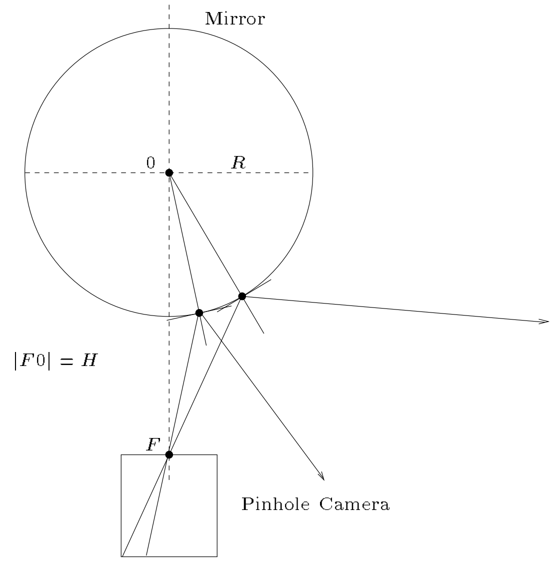

Figure 2. Escher's Mirror Geometry.

Therefore, we are led to the idea of using an "externally" coated parabolic surface for omniviewing. This requires that we arrange for a camera to look at the mirror with parallel rays, i.e. with orthographic – rather than perspective projection.

Indeed, looking at Figure 5, we see that a ray falling on the profile $y = f(r)$ vertically from below at $r = r_0$ will be sent into the world as if from a focal point F', $(0, H)$, if the following geometric condition will be satisfied:

$$\frac{\langle (r, f(r) - H),\ (1, f'(r)) \rangle}{\sqrt{r^2 + (f(r) - H)^2}} = \langle (1, f'(r)),\ (0,1) \rangle$$

leading to:

$$f'(r) = \frac{r + f'(r)[f(r) - H]}{\sqrt{r^2 + (f(r) - H)^2}} = \frac{d}{dr}\left(r^2 + (f(r) - H)^2\right)^{\frac{1}{2}} \ .$$

Hence

$$f(r) = \left(r^2 + (f(r) - H)^2\right)^{\frac{1}{2}} + \ \text{const}\,.$$

From this we easily obtain $f(r) = ar^2 + b$ is the general solution with $H = b + 1/4a$. Hence, given $f(0) = b$ and H we can determine the whole profile $f(r)$, and it is, not surprisingly, a parabolic profile. Suppose further that, for practical reasons, r is limited to $r_{\max}$ and we would like the rays at radius $r_{\max}$ to be sent into the world at an angle $\alpha_{\max}$. We then have

$$\tan\,\alpha_{\max} = \frac{f(r_{\max}) - H}{r_{\max}} = \frac{ar_{\max}^2 - 1/4a}{r_{\max}} \,,$$

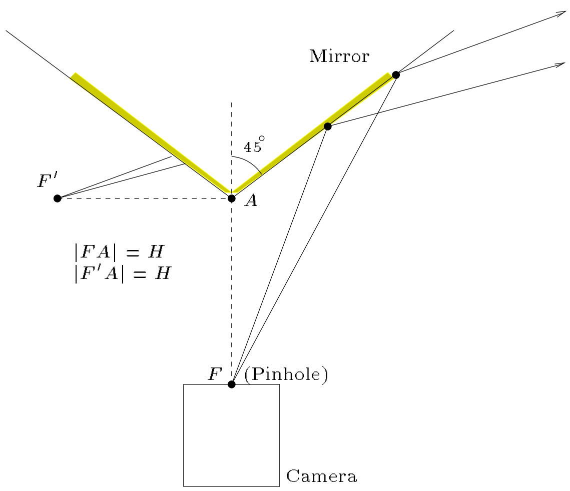

Figure 3. Conic Mirror Geometry.

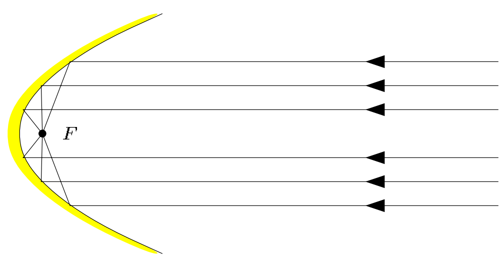

Figure 4. Focusing Rays with A Parabolic Mirror.

and this yields:

$$a = \frac{\sin\alpha_{\max} + 1}{2\,r_{\max}\cos\alpha_{\max}}\,.$$

If $r_{\max} = 1$ and $\alpha_{\max} = 45°$ we have $a = \frac{1+\sqrt{2}}{2} = 1.207$.

There are various ways to arrange for orthographic viewing. One could, for example, look at the mirror via an array of sensors located at the bottom of the vertical light guiding tubes, similar in structure to the eyes of some insects ("the composite eyes"). A particularly nice solution, however, is via the use of a second parabolic mirror, as shown in Figure 6. A pinhole camera looks down into a parabolic mirror, with the pinhole located at the focus F. This is equivalent to having an orthographic projection view of the parabolic mirror located above. The two mirror arrangement shown in Figure 6 achieves omniviewing via the use of a pinhole camera [BR95, BR96].

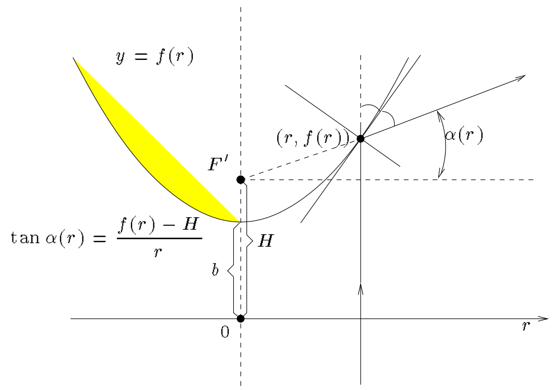

Figure 5. Omniviewing with Orthographic Projection leads to $f(r) = ar^2 + b$ mirror profile.

3.2. Omniviewing with Hyperbolic Mirrors

Our second idea for omniviewer design calls for the determination of a mirror profile that reflects the rays from a pinhole camera looking up, as if those would all originate without reflection from a common viewpoint F', on the mirror's symmetry axis. As in the previous section, we are looking for a mirror profile $y = f(r)$ for which, see Figure 7, the reflected ray $F'P$ and the incident ray FP form identical angles with the tangent to the profile at $P = (r, f(r))$. Letting $F' = (0, 1)$ and $F = (0, -1)$ this condition is equivalent to

$$\frac{\langle (r, f(r)-1),\ (1, f'(r))\rangle}{\sqrt{r^2 + (f(r)+1)^2}} = \frac{\langle (r, f(r)+1),\ (1, f'(r))\rangle}{\sqrt{r^2 + (f(r)+1)^2}}\,,$$

leading to

$$\frac{r + (f(r)+1)f'(r)}{\sqrt{r^2 + (f(r)+1)^2}} = \frac{r + (f(r)-1)f'(r)}{\sqrt{r^2 + (f(r)+1)^2}} \qquad (1)$$

or

$$\frac{d}{dr}|FP| = \frac{d}{dr}|F'P|\,.$$

From this we see that the curve $y = f(r)$ is the locus of points whose difference of distances to F and F' is constant. This locus is well known to be a hyperbola. More precisely, the solution to the differential equation (1) takes the form

$$f(r) = \sqrt{\frac{b}{1-b}\ r^2 + b}\,.$$

The parameter $b = f^2(0)$ is a design parameter that should be set to achieve desired performance in terms

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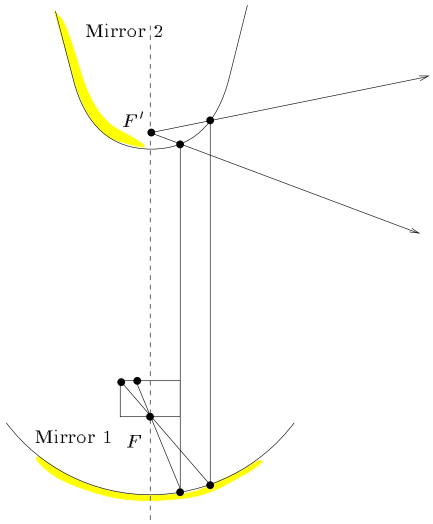

Figure 6. Omniviewer with Two Parabolic Mirrors.

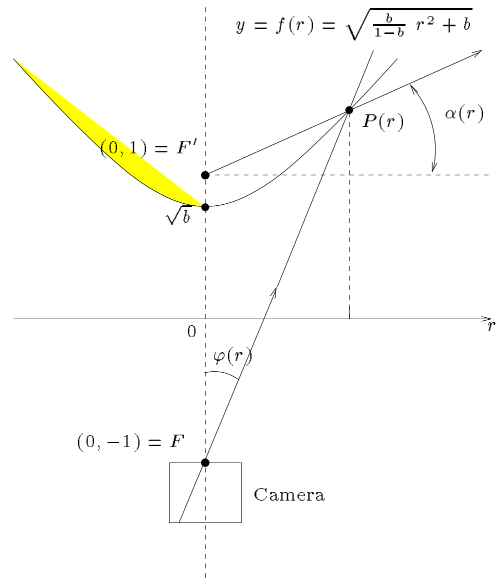

Figure 7. Hyperbolic Mirror Geometry.

of the spatial coverage of the mirror. To facilitate this we should express b in terms of r and α, or in terms of α and φ. Referring to Figure 7, we have

$$|FP| = |F'P| + 2\sqrt{b} \quad (2)$$
$$|FP|\sin\varphi = |F'P|\cos\alpha = r \quad (3)$$
$$|FP|\cos\varphi = |F'P|\sin\alpha + 2\,. \quad (4)$$

Squaring both sides of (3) and (4) and adding we obtain

$$|FP|^2 = |F'P|^2 + 4|F'P|\sin\alpha + 4\,.$$

If we now substitute for $|FP|$ using (2) then we have

$$4\sqrt{b}|F'P| + 4b = 4|F'P|\sin\alpha + 4\,. \quad (5)$$

If we now substitute $|F'P| = r/\cos\alpha$ (from (3)), then we obtain a quadratic equation in $\sqrt{b}$ which can be solved to obtain

$$\sqrt{b} = \frac{-r + \sqrt{2 + r^2 + 2\cos(2\alpha) + 2r\sin(2\alpha)}}{2\cos\alpha}\,. \quad (6)$$

Alternatively, we can solve (3) and (4) to obtain $|FP|$ and $|F'P|$ in terms of φ and α and then substitute the result into (2) to obtain

$$\sqrt{b} = \frac{\cos\alpha - \sin\varphi}{\cos(\alpha + \varphi)}\,. \quad (7)$$

Suppose that for $r = r_{\max}$ we wish to have a reflected (outgoing) ray at $\alpha(r_{\max}) = \alpha_{\max}$. We then apply equation (6) to obtain the required b. For example, setting $\alpha_{\max} = 45°$ and $r_{\max} = 1/2$ yields $b = 0.84861218$. This design is shown in Figure 8, [BrRi96].

From equation (5) we obtain

$$|F'P| = \frac{1-b}{\sqrt{b} - \sin\alpha}\,.$$

Dividing (4) through by (2) and substituting for $|F'P|$ from the above, we now obtain

$$\cos\varphi = \frac{2\sqrt{b} - (1+b)\sin\alpha}{(1+b) - 2\sqrt{b}\sin\alpha}$$

which can be used to determine the spacings in φ in order to sample the world over rings with equal spacing in α. This, as will be seen in the next section, is an important consideration in the design of cameras and sensor arrays for omniviewing.

4. Practical Considerations for Omniviewer Design

The previous sections addressed the problem of omniviewer design under idealized conditions: the assumptions were that an ideal pinhole camera is the imaging device and the sensor array generating the image has infinite resolution. Real life is not like this. A pinhole camera with very small aperture taking images on a photographic film would be a good approximation

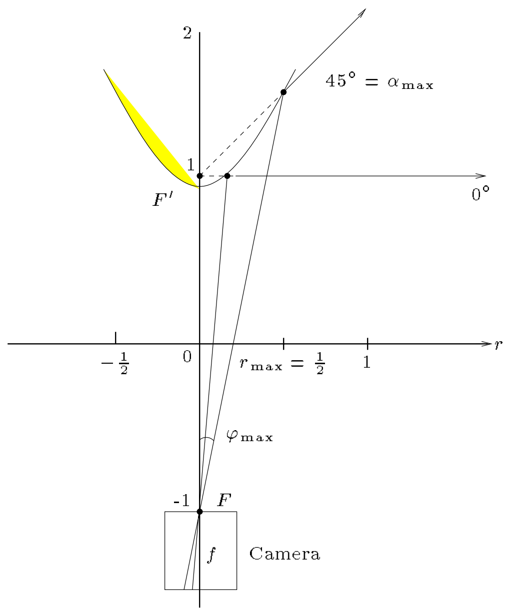

Figure 8. Hyperbolic Mirror Design.

of the idealized conditions described above. However, we wish to do omniviewing with a real camera having a high resolution CCD/or other type/of sensor array. This being the case we shall have to deal with the inevitable optical aberrations occurring when looking at curved mirrors through nonvanishing apertures and with issues of sampling in the image plane.

Under idealized conditions, the omniviewers proposed will produce an image $I(\theta, r)$, that will have to be mapped into a cylindrical image $I^D(x, y)$ via say:

$$I^D(x, y) = I(x = \theta,\ y = \lambda(r)),$$

where $\lambda(r)$ would be chosen so that y is linear in α. This mapping is quite problematic because we can readily see that the image resolution varies with r. Fortunately, of the image $I(\theta, r)$ produced by the camera we have to consider only an annular domain between r_{LOW} and r_{HIGH} only. Even this being the case, significant problems of resampling and interpolation necessarily arise if $I(\theta, r)$ is acquired with a square sensor array. We therefore propose to use in the camera a sensor array with sensor cells arranged in sets of concentric rings covering the area of interest only (i.e. the annular region between r_{LOW} & r_{HIGH}). The formulae developed in the preceding section relating the angle φ to the angle α of the rays looking into the world enable us to design the spacing of rings for the sensor array so as to correspond to a desired policy of sampling the world uniformly in the vertical direction. (For every α we shall sample the world over a ring of rays corresponding to all $\theta \in [0, 2\pi]$.)

Suppose that we want to sample the world from some $\alpha_{\min}$ to $\alpha_{\max}$ in equal spacings of $\Delta\alpha = \frac{\alpha_{\max} - \alpha_{\min}}{M}$. Then the rings in the sensor should correspond to the angles φ_i given by

$$\varphi_i = \arccos\left[\frac{2\sqrt{b} - (1+b)\sin(\alpha_{\min} + i\Delta\alpha)}{(1+b) - 2\sqrt{b}\sin(\alpha_{\min} + i\Delta\alpha)}\right]$$

and their space locations will be determined by

$$r_i = f \cdot \tan\varphi_i \quad (f \text{ - focal length}).$$

Hence when designing a camera and sensor we have the formulae for the complete design of the circular sensor array in terms of the mirror design (b) the spatial coverage ($\alpha_{\max}, \alpha_{\min}$) and sampling rate ($M$) and the focal length ($f$) of the camera. We should aim for high resolution both radially (θ) and vertically (M).

A potentially more serious problem that arises in connection with the proposed mirror based omniviewer is the effect on aberration due to curved reflecting surfaces. The aberrations will be necessarily present because we shall use a camera with lenses and finite aperture. This means that in each direction there will be several rays (i.e., a beam of rays) (parallel if the camera focuses at infinity) mapped into each point on the sensor array. The curved mirror will cause the beam of rays to diverge giving rise to a "depth dependent" space varying blurring effect. This blurring effect can be reduced by making the aperture smaller, but this, in turn will decrease the amount of light gathered by the camera, leading to problems of dynamic range, and diffraction too [Fr90].

We are in the process of testing these solutions and examining ways to overcome the problems of aberrations and sampling effects either via the design of special purpose sensor arrays and cameras or by some image sharpening algorithms.

5. Concluding Remarks

We have presented several ideas for the design of "true" omnidirectional cameras, with the help of parabolic and hyperbolic mirrors. These are currently being tested for the possible implementation of omniview cameras. The applications of such devices are many. In the past such devices were proposed in the area of robotics, for navigation and self location (see the "COPIS" references [YKT94]). We wish to emphasize the important teleconferencing application. We imagine an Omniview Device built to look like a rather small table lamp, as the video-input device for a versatile teleconferencing system. It could generate the output on a cylindrical TV projection device, to show the surrounding scene at the other end of the communication system, or it could generate the desired section of the image on a flat screen (concentrating on the speaker only, say). All this could be done without moving cameras.

Also, as pointed out by Nalwa, the same ideas can be applied to the building of an omnidirectional projection system, the camera being replaced by a projection device.

6. Acknowledgements

Thanks to Vic Nalwa for many important discussions on this topic and his help and encouragement. Thanks to Bob Holt for help with some early MAPLE calculations for the design of Figure 7.

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