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On Image Extrapolation

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ON IMAGE EXTRAPOLATION

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ABSTRACT

An image extrapolation procedure is presented, enabling us to see the whole picture, when only part of it is available.

1. INTRODUCTION

It is a well known result of advanced calculus, see e.g. [1], that an analytic bivariate function is completely determined if we have its values over any compact region of the plane. Indeed, we can calculate all the derivatives of the function at a point in the given region, then write its Taylor expansion providing the values of the function everywhere. This is quite straightforward, and it could seem that this result trivializes the image extrapolation problem. This, however, is not the case. As practical people dealing with the processing of real images, we know that

(1) images are not always analytic functions and

(2) we are usually dealing with digitized images, resulting from sampling and quantization of the continuous distribution of luminance and color.

These practicalities seem to be formidable roadblocks on our way to develop robust, efficient and well conditioned extrapolation procedures. Therefore, it was no little surprise for the author of this note to discover, while browsing in the cellars of the University Library, a brilliant but completely overlooked and underevaluated but seminal paper by Lobachevsky and Krasnolselsky, published in 1837 in the Fourth Series of the Syberianskaya Gazetta Mathematicorum, [2], dealing with "approximate topological and analytical extrapolation of nonanalytic sampled bivariate distributions", a subject surely found at the heart of our problem.

2. TESTING THE EXTRAPOLATION ALGORITHM

A few weeks of intense study of the Lobachevsky-Krasnolselsky paper provided the first practical extrapolation procedure (I hereby propose to call it the L-K algorithm) that yielded reasonable extrapolations. The image I chose to submit to the extrapolation algorithm is the face of a woman, shown in Picture 1. This is the standard input used to test image processing algorithms everywhere. The result of extrapolating the image is shown in Picture 2. We can see that the straightforward algorithm is quite crude, and the extrapolation degrades very quickly as we proceed further from the given data. Since we tested the algorithm in one direction only, and this direction was chosen (without loss of generality but with an obvious choice for the region of interest) to be SE-downward, we may call these procedures "downward continuation algorithms", a name borrowed from geophysics and used there for algorithms reconstructing the layers of the earth from seismic exploration data, see e.g. [3]. As we see, the straightforward application of the L-K procedure provided only very crude results. Then I proceeded to improve the process and stabilize the extrapolation, via a standard Tichonov regularization method, [4]. This yielded Picture 3. Here we can much better distinguish what is going on, however the result is still quite fuzzy and blurred. Then I decided to apply a directional filtering technique due to Gabor, [5], the father of holography, to further improve the propagation of the downward continuation process. This time the extrapolations were considerably improved, and we were in fact a bit surprised by the way the image was developing downward. The result is shown in Picture 4.

3. CONCLUDING REMARKS

At this point my interest in the results of downward extrapolation algorithms was much aroused and I started researching the history of the input picture, to see whether I can get an independent assessment of the quality of my extrapolation results. After showing my results to various people, a learned colleague of mine who treasures a complete collection of the Playboy magazine, and also has a formidable memory, recalled that a certain lady from Sweden, did some artistic pictures for the magazine in the seventies, and her centerfold looked similar to my reconstructions. We eagerly pursued our research in this interesting direction (funded by various governmental agencies) and came up with the November 1972 issue of the

above-mentioned learned publication, [6]. In it we found the centerfold shown here as Picture 5, of Ms. Lenna Sjooblom, from the village of Jarna in Sweden.

It is quite clear that the above findings fully support my claims of having developed a reliable, stable and effective image extrapolation process. In order to test the result further, I took the head of Ms. Michelle Pfeifer, alias Catwoman, from a frame of the recent motion picture titled BatmanII, and applied the extrapolation process to this picture, in order to recover the full image shown in Picture 6. The result was Picture 7, not perfect yet but clearly encouraging.

Due to the obviously enormous potential for applications, I chose not to disclose here the precise algorithm. It is now being patented worldwide, in spite of the advice of my lawyers not to proceed at all with the disclosure involved in patenting such a potentially lucrative procedure.

4. ACKNOWLEDGEMENTS

It is my pleasure to thank Mr. Roni Kimmel for his help in implementing the extrapolation procedures, and the members of the Technion PIXEL CLUB, for their constant encouragement and interest in this line of research.

5. REFERENCES

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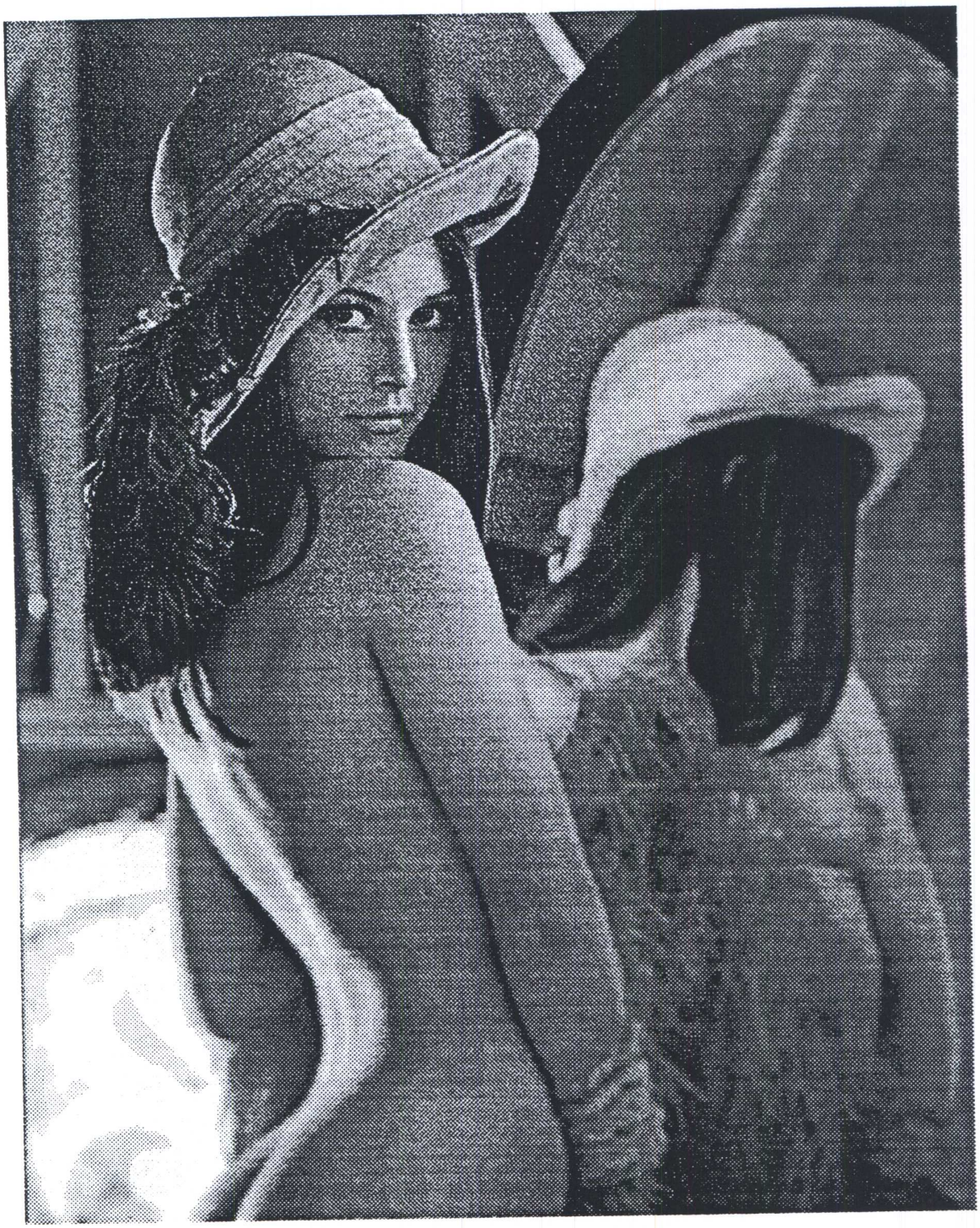
[6] Playboy Magazine, "The Playmate of the Month", November 1972.

PICTURE 1.

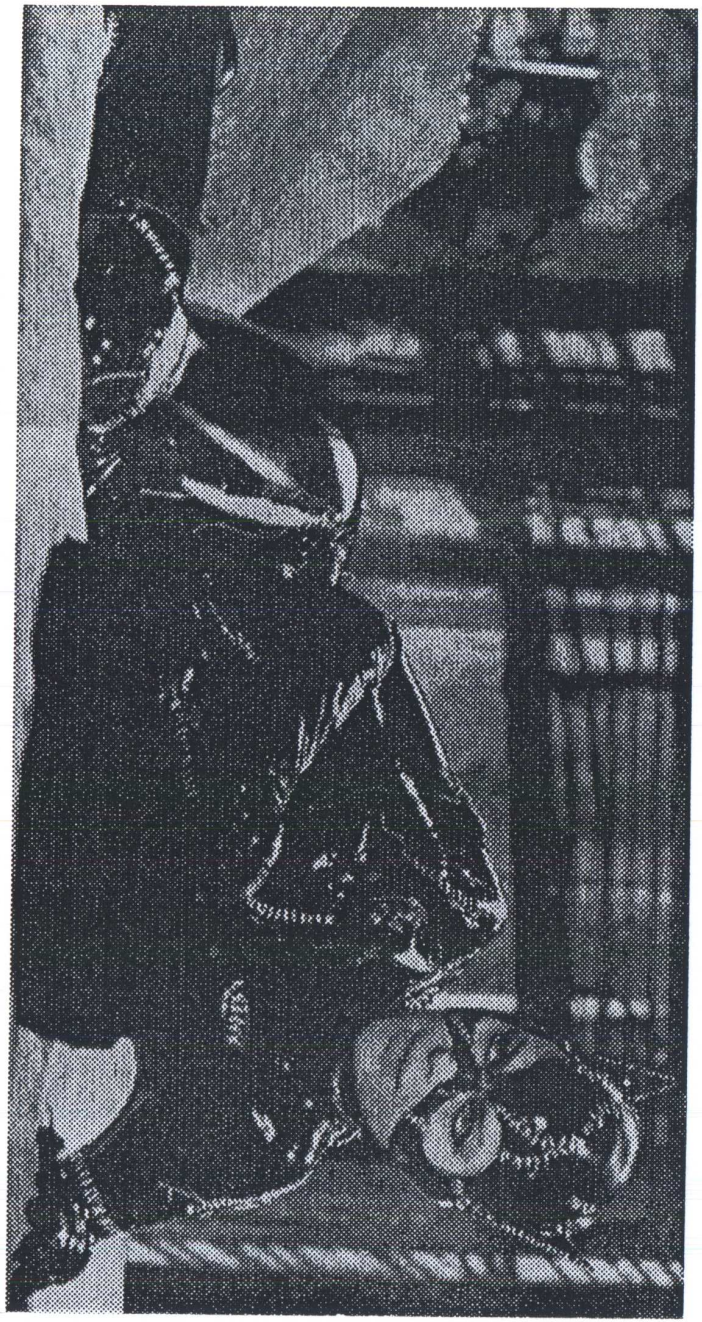
PICTURE 2.

PICTURE 3.

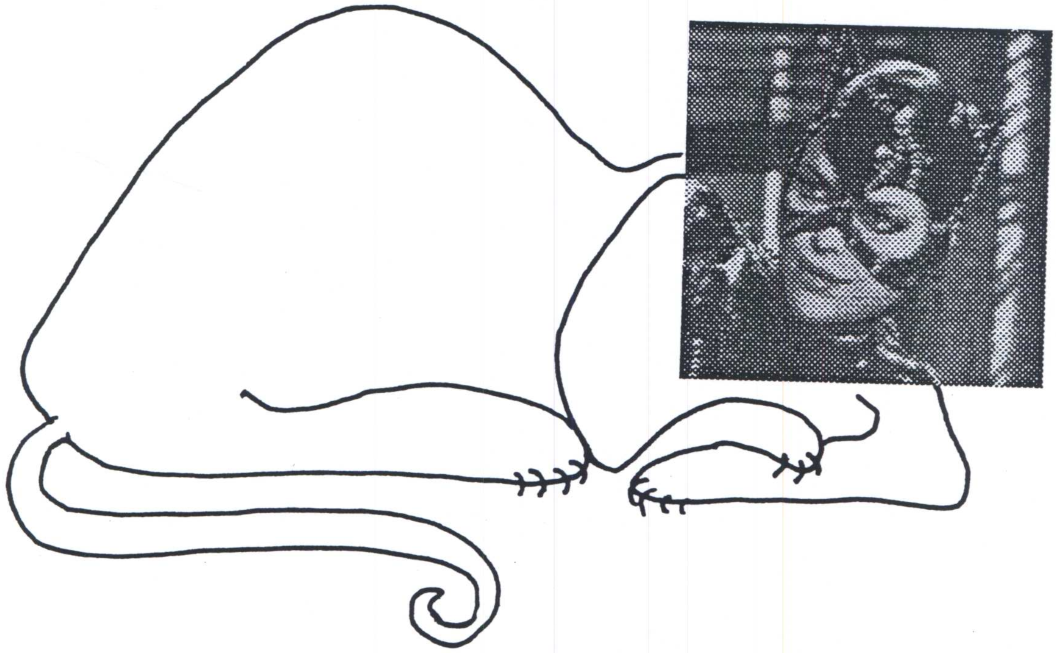


PICTURE 4.

PICTURE 5.



PICTURE 6.



PICTURE 7.