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ON GABOR'S CONTRIBUTION TO IMAGE ENHANCEMENT †

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Abstract—Dennis Gabor is mainly known for the invention of optical holography and the introduction of the so-called Gabor functions in communications. A few people know that he was also interested in image processing. In a paper entitled "Information theory in electron microscopy" (*Laboratory Investigation* **14**(6), 801–807 (1965)), written in 1965, he examined the problem of image deblurring and was the first to suggest a method for edge enhancement based on principles widely accepted today and implemented in advanced image processing systems. In this paper his ideas are reviewed, their relation to contemporary methods is shown, and some simulations he could not do in 1965 are performed.

Edge detection   Image enhancement   Directional filtering   Anisotropic diffusion

INTRODUCTION

Several modern techniques for image enhancement and edge detection have a lot in common with an early idea of Dennis Gabor, winner of the Nobel prize in physics for the invention of optical holography and also famous for the introduction of the so-called Gabor functions in the theory of communication. However, very few papers in the image processing literature mention Gabor's idea, perhaps because it appeared in the relatively obscure *Journal of Laboratory Investigation*, published by the International Academy of Pathology. Gabor's paper, "Information theory in electron microscopy",[1] hides under this title the proposal for the first image deblurring method based on directional sensitive filtering.

Gabor claims that his method is information theoretic in the wide sense of making "optimum use of a certain type of prior information". After deriving the enhancement formulae (to be discussed and reviewed in the sequel) he claims that: "with modern methods there would be no difficulty in feeding the image values into a computer [*sic*!] which instantaneously computes the necessary values and feeds them into the CRT".

In a 1970 review of image enhancement methods, Judith Prewitt mentioned Gabor's ideas.[2] However, the next and, so far, only additional reference to this work in the image processing literature appears in a 1983 survey paper by Wang *et al.*[3] The latter authors, just like Prewitt before, show Gabor's formulae mentioning that the computation involved is complicated. It may therefore seem that from 1965 to 1983, and even until today, there has not been much progress in our technical capabilities of implementing the methods proposed by Gabor. Of course, this is not true. What did happen was that ideas similar to Gabor's led to a wealth of modern, highly sophisticated methods of local and adaptive image enhancement, edge detection and segmentation. Few people, however, know and credit Gabor's proposal of adaptive directional filtering methods that seem to be the modern trend in image processing.

It is the aim of this paper to review Dennis Gabor's ideas, and restore some credit to this highly original researcher whose work came too early to be appreciated.

DEBLURRING OF IMAGES

Blurring usually occurs as a result of imperfections at the image acquisition stage. A common model for the process of blurring assumes that it is equivalent to filtering the image through a two-dimensional (2D) linear system whose impulse response, h, is a Gaussian function. The blurred version is given by

$$g(x, y) = \int h(x - \eta, y - \xi, \sigma) f(\eta, \xi) \, d\eta \, d\xi \qquad (1)$$

where

$$h(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2 + y^2)/2\sigma^2} \qquad (2)$$

and the parameter σ controls the amount of blur. This blurring operation may also be viewed as the result of a 2D diffusion process determined by the heat equation

$$\frac{\partial \phi}{\partial t} = \nabla^2 \phi \qquad (3)$$

defined on an infinite domain, and with initial condition $\phi(x, y, t = 0) = f(x, y)$. Since the heat equation (3) is solved by

$$\phi(x, y, t) = \int h(x - \eta, y - \xi, t) \phi(\eta, \xi, 0) \, d\eta \, d\xi \qquad (4)$$

† Dedicated to the memory of D. Gabor, 13 years after his death.

the kernel $h(x, y, t)$ being the following function of t

$$h(x, y, t) = \frac{1}{4\pi t} e^{-(x^2+y^2)/4t}$$

we see that $\phi(x, y, t)$ is identical to $g(x, y)$ provided $\sqrt{(2t)} = \sigma$. Methods for deblurring, i.e. for restoring the original image f, from its blurred image, g, have drawn considerable attention (see reference (4) for a recent survey). Clearly, this inverse problem does not have an exact solution and is also difficult to solve numerically as it is clearly ill-posed. In the inevitable presence of noise the deblurring problem is even more severe, and many methods, including inverse filtering, Wiener filtering, and iterative schemes were proposed to solve it.[4,5]

If the amount of blurring is small, some simple methods are quite effective. Since low blurring may be viewed as a result of diffusion for a short time τ, we have that

$$\phi(x, y, 0) = \phi(x, y, \tau) + (-\tau)\frac{\partial \phi(x, y, t)}{\partial t}\bigg|_{t=\tau} + O(\tau^2). \quad (5)$$

Hence, for small τ, $\phi(x, y, 0)$ may be approximated by taking only the first two terms in the above Taylor expansion. Then, using equation (3), the approximation of the initial condition becomes

$$\phi(x, y, 0) = \phi(x, y, \tau) - \tau \nabla^2 \phi(x, y, t)|_{t=\tau}. \quad (6)$$

This yields the approximate deblurring equation

$$\hat{f}(x, y) = g(x, y) - C\nabla^2 g(x, y) \quad (7)$$

where C is a constant to be determined empirically. Hence, for a small amount of blur, without noise, combining the image with its Laplacian is the optimal restoration filter. This method was introduced to image processing by Kovasznay and Joseph, who even implemented it using analog circuitry affecting the video signals in real time.[6] Gabor suggested a practical heuristic to determine the unknown constant C: its value should be chosen so that the steepest slope in the image is doubled. In this case the maximal overshoot is 5%, and no visible image distortion will arise.

GABOR'S FIRST METHOD FOR IMAGE ENHANCEMENT

Gabor considered the Laplacian method of Kovasznay and Joseph, and observed that the rotationally invariant Laplacian magnifies the noise equally in all directions. Therefore, while improving the steepness of edges it also enhances their "jaggedness". Hence he suggested to substitute the isotropic Laplacian by a directional operator, and to deblur the image using

$$\hat{f}(x, y) = g(x, y) - C\frac{\partial^2 g}{\partial n^2} \quad (8)$$

where n is the coordinate in the direction of the gradient. Gabor observed that the explicit expression of the directional second derivative

$$\frac{\partial^2 g}{\partial n^2} = \frac{\frac{\partial^2 g}{\partial x^2}\left(\frac{\partial g}{\partial x}\right)^2 + 2\frac{\partial^2 g}{\partial x \partial y}\frac{\partial g}{\partial x}\frac{\partial g}{\partial y} + \frac{\partial^2 g}{\partial y^2}\left(\frac{\partial g}{\partial y}\right)^2}{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2} \quad (9)$$

is too complicated to implement using the kind of analog circuitry used by Kovasznay and Joseph for the isotropic Laplacian, and thus suggested to scan the image with a special "probe" consisting of nine local intensity sensors and to process their output using a computer. Expression (9) is derived by defining a coordinate system whose axes are in the direction of the local image gradient and the tangent vector. Evaluating $\partial^2 g/\partial n^2$ then leads to the bilinear matrix form

$$\frac{\partial^2 g}{\partial n^2} = [n_x \; n_y]\begin{bmatrix} \frac{\partial^2 g}{\partial x^2} & \frac{\partial^2 g}{\partial x \partial y} \\ \frac{\partial^2 g}{\partial x \partial y} & \frac{\partial^2 g}{\partial y^2} \end{bmatrix}\begin{bmatrix} n_x \\ n_y \end{bmatrix} \quad (10)$$

and inserting the expression for the normal components yields expression (9).

Choosing a coordinate system attached to a particular edge with axes in the direction of the normal to the edge and the tangent to it, the Laplacian is also given by

$$\nabla^2 g = \frac{\partial^2 g}{\partial n^2} + \frac{\partial^2 g}{\partial s^2}. \quad (11)$$

Hence, Gabor's proposal is simply to keep only the first term of the Laplacian and ignore the second one, as the latter does not enhance the edge but only causes magnification of noise.

GABOR'S SECOND IMPROVEMENT

Gabor extended his idea further and suggested not only to delete the double differentiation in the tangent direction but also to smooth, i.e. integrate the image in this direction. Under the assumption of a straight edge, this averaging has the desired effect of decreasing noise. With this averaging, the deblurring operator becomes

$$\hat{f}(x, y) = g(x, y) - C\left(\frac{\partial^2 g}{\partial n^2} - \frac{1}{3}\frac{\partial^2 g}{\partial s^2}\right). \quad (12)$$

In his paper, Gabor did not explain how he derived this expression, but he probably used the following simple argument. Consider the cross section $g_{cs}(s)$ of the 2D function $g(x, y)$ along a line that passes through the point (x_0, y_0) in the tangent direction s. The diffusion expands each point to a gaussian with a standard deviation of $\sqrt{(2\tau)}$, and thus it makes sense to integrate this function over a span of $2\sqrt{(2\tau)}$. Approximating $g_{cs}(s)$ by the first three terms of its Taylor series and calculating the integral, we get that the averaging is equivalent to adding the second derivative $\partial^2 g/\partial s^2$ term multiplied by $\tau/3 = C/3$. The explicit expression for the derivative $\partial^2 g/\partial s^2$ is similar to expression (9)

$$\frac{\partial^2 g}{\partial s^2}=\frac{\dfrac{\partial^2 g}{\partial x^2}\left(\dfrac{\partial g}{\partial y}\right)^2-2\dfrac{\partial^2 g}{\partial x\,\partial y}\dfrac{\partial g}{\partial x}\dfrac{\partial g}{\partial y}+\dfrac{\partial^2 g}{\partial y^2}\left(\dfrac{\partial g}{\partial x}\right)^2}{\left(\dfrac{\partial g}{\partial x}\right)^2+\left(\dfrac{\partial g}{\partial y}\right)^2}. \tag{13}$$

Gabor claimed that such a correction will further reduce the jaggedness of the edge, and stated his belief that the edge detection apparatus in the human visual system operates according to similar principles. Note that this correction term is equivalent to performing a directional diffusion smoothing of the type

$$\frac{\partial f}{\partial t}=\frac{\partial^2 f}{\partial s^2} \tag{14}$$

for a short while.

Since we know that adding the Laplacian is in some sense an optimal deblurring filter, one may question why the Gabor method should perform better. A qualitative argument may be given based on the theory of least squares restoration. The optimal linear filter for restoring a noisy and distorted signal, known as the Wiener filter, is a compromise between good distortion correction and robustness to noise. In the Wiener filter framework, both the signal and the noise are assumed to be random processes, and are characterized by their power spectra. Qualitatively, in regions where the power spectrum of the image is much stronger than the spectrum of the noise, the Wiener filter approximates the inverse filtering operation, that should restore the image exactly (but magnifies the additive noise). On the other hand, spectral regions where the spectrum of the image is weaker than the spectrum of the noise, are filtered out or attenuated. Assume that the image is a sample of a random process of images that vary only along one fixed direction. Such are images that contain only a single edge in a given direction, whose slope and shape may vary. The power spectrum of such a process is zero everywhere but (possibly) on one line in the Fourier domain, passing through the origin and normal to the edge direction (see Fig. 1). Suppose this simple "one edge" image has undergone some blurring and was also corrupted with additive white noise. The frequency domain transfer function of the Wiener filter for restoring this image should therefore be zero everywhere except possibly on the above described line.

Both the Laplacian method of Kovasznay and Joseph and the first method suggested by Gabor lack this property, since their outputs include the noisy image itself whose spectrum, due to the noise, is nonzero everywhere. However, smoothing the image in the direction of the tangent, as done in Gabor's second method, implies cutting off high frequencies in this direction. Hence the frequency response of this filter may be high only for frequencies located on a line with direction perpendicular to the edge, decreasing for frequencies distant from this line. The wider the averaging along the direction of the edge, the more concentrated near this linear region becomes the corresponding frequency response. Thus, Gabor's second method locally approximates a desired property of the "optimal" filter. This stands in contrast to the transfer function of the Laplacian that is isotropic in the Fourier domain and certainly not restricted to the required frequency band or to its neighborhood. In fact, it even increases for frequencies distant from the band. Gabor's first method stands between the other two with respect to this property. Here, the value of the transfer function neither decreases nor increases for frequencies distant from the band.

A FURTHER IMPROVEMENT

When the blurring becomes more substantial, and the corresponding effective diffusion distance $\surd(2t)=\sigma$ becomes higher, adding the second directional derivative $\partial^2 g/\partial s^2$ is no longer a good approximation for image averaging in the tangent direction. Adding substantial noise makes the image less smooth and also contributes to this inaccuracy. A simple way to overcome this difficulty, which we shall call the Modified Gabor method, is to carry out the averaging in a sequence of identical stages, each consisting of calculating the directional derivatives of the smoothed image and adding to it the second directional derivative $\partial^2 g/\partial s^2$

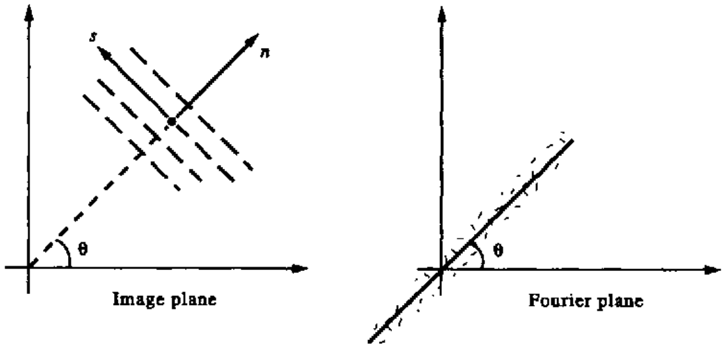

Fig. 1. A straight edge and its frequency representation.

multiplied by a small constant. Each such stage of the process approximates the averaging of the image over a small region in the tangent direction, and repeating this procedure several times is equivalent to averaging the image over an extended region with a locally image dependent weighting function. Here, the basic averaging regions are small, and therefore Gabor's approximation is valid for each stage, resulting in a good approximation to averaging in the extended region too. After the directional averaging is done the second directional derivative $\partial^2 g/\partial n^2$ (of the averaged image) is subtracted, as in Gabor's first method. The Modified Gabor method improves the performance of the original methods, especially when the local approximation of the image using the Taylor series requires many terms, for example when curvature of the image edges is high or when the added noise is substantial.

DIRECTIONAL BLURRING

Although subtracting the weighted Laplacian is the optimal restoration filter for a noise free image degraded by a diffusion process, Gabor's suggestions should perform better when noise is present. Note that if the blurring is equivalent to a short time, noiseless, directional diffusion process

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial n^2} \tag{15}$$

Gabor's first method would indeed be the optimal restoration filter. For general inputs, isotropic diffusion (3) and directed diffusion (14) may be very different, but for images composed mainly of relatively long edges, these two blurring processes are quite similar. We found in simulations that the r.m.s. difference between two such diffused versions of an image is much smaller (by 22 db), than the difference between each diffused image and the original. Nevertheless, we tested the enhancement procedures on images distorted by both diffusion processes and found that all methods performed nearly equally on both types of inputs.

EXPERIMENTAL RESULTS

Usually a deblurring process comprises two stages. In the first stage, one estimates the amount of blurring and, in the second stage, deblurring is performed based on the estimate obtained. Gabor was interested only in the second stage, and assumed that the blurring process, i.e. the equivalent diffusion time span is known.

The main objective of our experiments was to test Gabor's original claims empirically. Two images were used in the simulations. The first one was the classical 256 × 256 "Lena" image. The second one was a synthetic image comprising 64 circles of different radii and contrasts (see Fig. 2).

The images were diffused by either isotropic or directional diffusion. The former was done by convolving

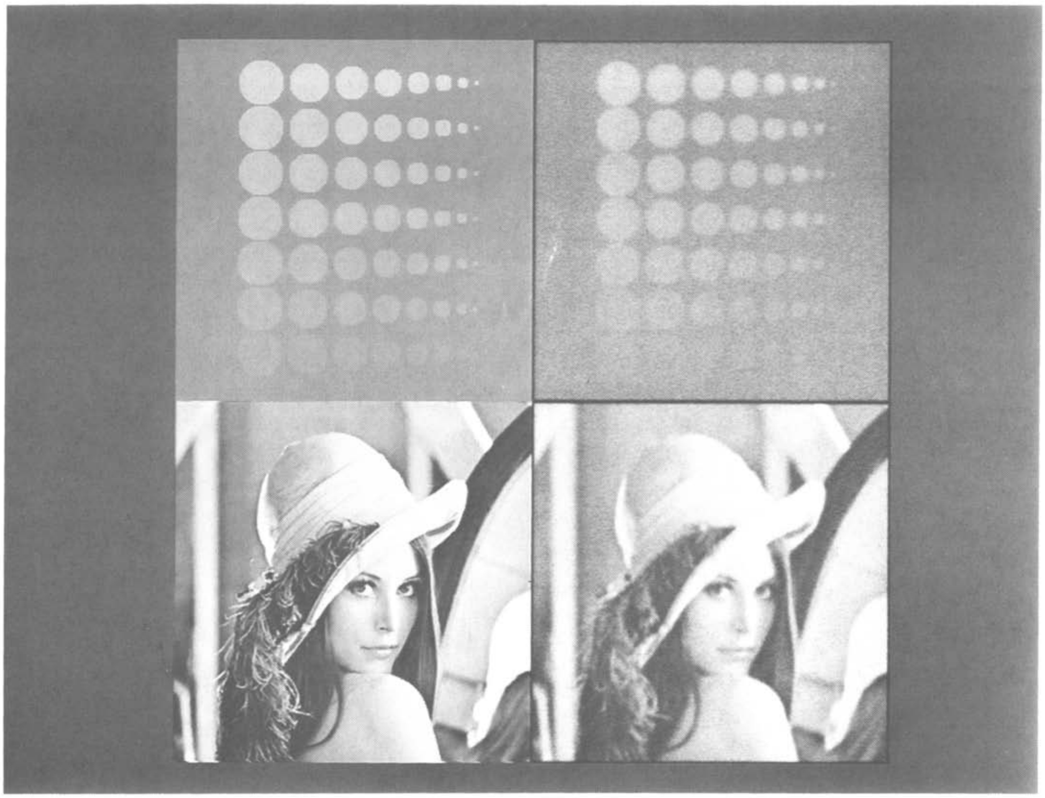

Fig. 2. The original test images and their blurred, noisy, versions. The distorted images were created by eight convolution steps followed by adding uniform noise with amplitude 11.

image several times with the mask

$$\frac{1}{12}\begin{vmatrix} 0 & 1 & 0 \\ 1 & 8 & 1 \\ 0 & 1 & 0 \end{vmatrix},$$

equivalent to a diffusion process operating for 1/12 time units. The latter was done simply by adding the second directional derivative at each step (multiplied by 1/12). After distorting the image, uniform white noise was added. The two test images with and without isotropic blurring are shown in Fig. 2. (Noise was added to the blurred images.)

Table 1

Test image	Number of blurring steps	Noise amplitude	Input SNR: SNR_n	Input SNR: SNR_{nd}	Output SNR: Laplacian method	Output SNR: Gabor's first method	Output SNR: Gabor's second method	Output SNR: Modified Gabor method
Circles	8	0	∞	5.5	10.6	10.5	10.4	10.1
		5	10.9	4.3	−0.6	2.6	3.8	6.9
		11	4.14	1.72	−7.42	−3.66	−2.15	2.5
	32	0	∞	−0.4	2.2	2.2	2.2	2.2
		5	10.9	−0.8	−11.1	−6.4	−4.6	−1.3
		11	4.14	−1.7	−17.2	−12.7	−10.7	−5.9
"Lena"	8	0	∞	9.9	10.8	10.9	10.9	10.8
		5	24.7	9.9	9.0	10.1	10.3	10.1
		11	17.9	9.4	5.1	7.6	8.4	10.0
	32	0	∞	7.1	8.7	8.6	8.6	8.6
		5	24.7	7.1	2.00	5.3	6.2	7.7
		11	17.9	6.8	−3.4	0.6	2.2	5.4

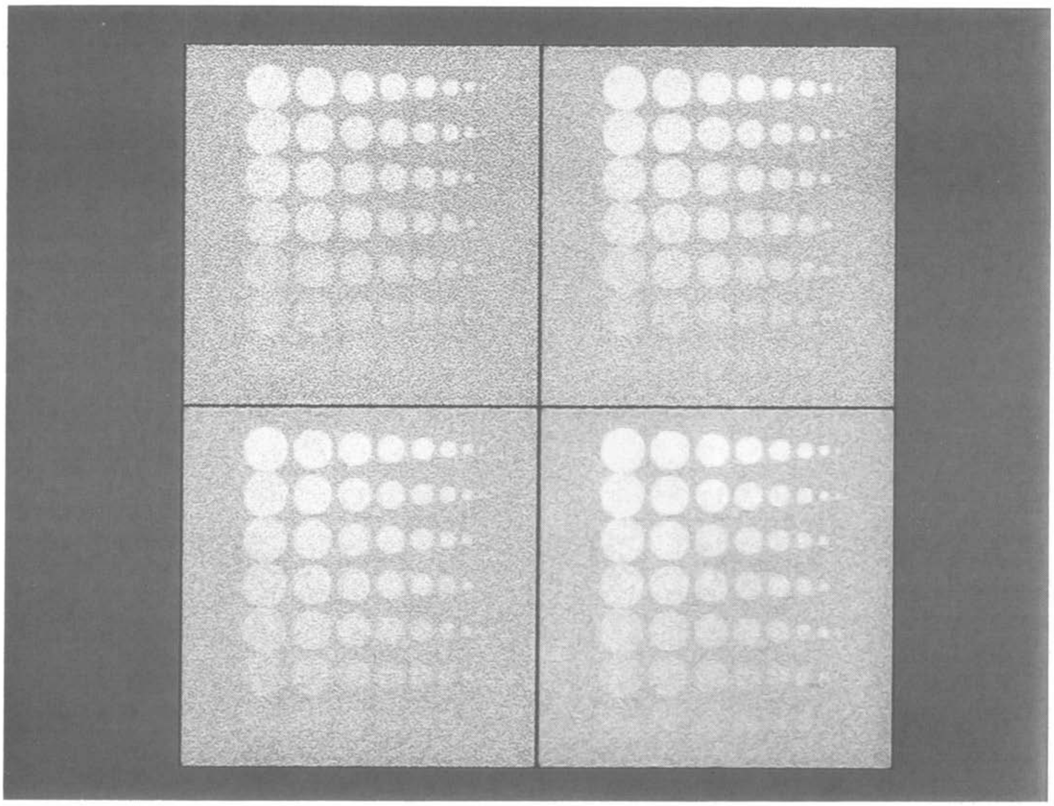

Fig. 3. A comparison between the performance of the four enhancement methods on the distorted "circles" image. The upper left image shows the result of deblurring using the Laplacian method, the upper right image shows the result of using Gabor's first method, the lower left image shows the result of Gabor's second method and the lower right image shows the result of deblurring with the Modified Gabor method. The original image is blurred using eight steps of isotropic diffusion, and uniform noise of amplitude 11 is added. The output SNRs are −7.42, −3.66, −2.15, and 2.5, respectively.

Two types of input signal to noise ratio are used to characterize separately the contribution of the noise and the diffusion to distorting the image. Their definitions are

$$\mathrm{SNR}_n = 20\log(\text{r.m.s. [original image]}/\text{r.m.s. [added noise]})$$

$$\mathrm{SNR}_{nd} = 20\log(\text{r.m.s. [original image]}/\text{r.m.s. [difference image]})$$

where the difference image is the result of subtracting the original image from its distorted and noisy version.

We tested the four deblurring methods discussed above:

(a) The Laplacian method as described by expression (7).

(b) Gabor's first method as described by expression (8).

(c) Gabor's second method as described by expression (12).

(d) The Modified Gabor method—a deblurring method extending Gabor's ideas as explained in this paper. Instead of averaging the image in the tangent direction, we followed Gabor's suggestion and improved it by adding the second directional derivative (13) in this direction. Then the second directional derivative in the gradient direction was subtracted (as in Gabor's first method).

We also implemented a variation on Gabor's methods, in which the estimation of the local gradient direction is calculated from a smoothed version of the image. The results were not significantly different from the original suggestions and are not discussed here.

The performance of each of these methods was examined both visually and quantitatively for various amounts of blurr and noise. Quantitative results are summarized in Table 1. The deblurred image SNR is defined by

$$\mathrm{SNR}_{out} = 10\log(\text{r.m.s. [original image]}/\text{r.m.s. [difference image]})$$

and here the difference image is the difference between the original image and the deblurred image. The differences in the performance of the methods tested are visible in Figs 3 and 4, showing the deblurred images. In each quarter of these figures, the upper left image shows the result of deblurring using the Laplacian

Fig. 4. A comparison between the performance of all methods on the distorted "Lena" image. The upper left image shows the result of deblurring using the Laplacian method, the upper right image shows the result of using Gabor's first method, the lower left image shows the result of Gabor's second method, and finally, the lower right image shows the result of the deblurring with the Modified Gabor method. The original image is blurred using eight steps of isotropic diffusion, and uniform noise of amplitude 11 is added. The output SNRs are 5.1, 7.6, 8.4 and 10.0, respectively.

method, the upper right image shows the result of using Gabor's first method, the lower left image shows the result of Gabor's second method, and finally, the lower right image shows the result of the deblurring with the Modified Gabor method. For these images, the diffusion time was 8 steps equivalent to 0.66 time units, hence taking only the first term of the Taylor expansion as an approximation is not strictly justified. Lower diffusion times resulted in image distortions that were not readily perceptible and in order to get the opportunity to judge the results visually, we chose to present images with relatively high diffusion times.

The size of the averaging interval and the number of local averaging steps in the Modified Gabor method should preferably depend adaptively on the local curvature of the edge. However, we did not implement this improvement and always used five averaging steps, each one performed over a fixed interval equal to the diffusion distance corresponding to one blurring step. The experiments reveal clearly that Gabor methods perform better than the Laplacian, their advantage becoming greater for lower signal to noise ratios. As expected, for very low input SNR, the performance of the Modified Gabor method outperforms even the second method of Gabor.

DISCUSSION

The main objective of this paper was to discuss Gabor's contribution to image enhancement and examine experimentally some of his early proposals for image deblurring. The results demonstrate that the approaches he suggested indeed outperform the well-known Laplacian method. As expected, the performance of the methods tested depended on the input signal to noise ratio. When the input SNR is very high, the Laplacian is the best restoration filter, as the directional (nonisotropic) methods degrade near high curvature edges. However, when noise is added and the signal to noise deteriorates, Gabor's methods which do not amplify the noise as much as the Laplacian, perform better. Finally, when the SNR is very poor, one can benefit from more substantial directional averaging, which is not provided by Gabor's method and it is here where the Modified Gabor method becomes the winner.

Gabor's proposals may seem rather straightforward today. This is simply because we are already aware of more recent research based on these principles. Canny's edge detector,[7] for example, is also based on the directional filtering principle and on smoothing the image in the tangent direction. One problem with Gabor's methods is the smoothing of the image in only one direction, even when the image is locally nearly isotropic. This may cause spurious edges. A clever modern technique that solves this problem is the so-called anisotropic diffusion procedure proposed by Perona and Malik. This method improves the image by smoothing it only where the gradient is small, with preference to averaging perpendicular to the gradient direction.[8] A rudimentary version of this idea appears in an early paper of Graham, describing a real-time image denoising hardware.[9] For a series of very interesting further developments of the topic of image enhancement using adaptive diffusion processes see references (10–13).

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