JOURNAL OF VISUAL COMMUNICATION AND IMAGE REPRESENTATION
Vol. 6, No. 1, March, pp. 52–58, 1995

Bit Allocation in Piecewise-Planar Representation of Images

NAHUM KIRYATI,*,[1] ALFRED M. BRUCKSTEIN,† AND AMNON JONAS*

*Electrical Engineering Department and †Computer Science Department, Technion—Israel Institute of Technology, Haifa 32000, Israel

Received July 22, 1992; accepted August 24, 1993

It is customary to represent an analog image in digital form by dividing its support into pixels, and within each pixel to represent the brightness by a quantized scalar, i.e., to approximate the 2-D image function by a horizontal plane. A representation scheme in which the image function is approximated within a pixel by a sloped plane, given in terms of the average and the slopes in both spatial coordinates, is considered. The problem of optimal bit allocation for the mean and the slope coefficients is studied for deterministic images and for samples of 2-D random processes with known autocorrelation functions. Similar results are available for the 1-D case. © 1995 Academic Press, Inc.

1. INTRODUCTION

In order to enable the processing of an image by a digital computer, the image is usually represented by a matrix of discrete values. To represent an image function $f(x, y)$ defined on, say, $\Omega = [0, 1] \times [0, 1]$, Ω is typically divided to $M \times N$ picture elements (pixels), and within each pixel the image is described by a scalar that corresponds either to the average of $f(x, y)$ within the pixel or to the value of $f(x, y)$ at the center of the pixel. These scalars are subsequently quantized to obtain an array of $M \times N$ discrete values. With b bits per pixel, this digital approximation of $f(x, y)$ requires a total of $B = M \times N \times b$ bits.

There is a clear trade-off between the similarity of $f(x, y)$ to its digitized approximation (typically defined in terms of square distance) and the volume B of the representation. Furthermore, given B bits, a trade-off arises between the spatial resolution (by M and N) and the quantization depth (by b). Optimal representation of a given bivariate image function $f(x, y)$ was determined in [6] for the case of point sampling and zero-order hold interpolation. Given B, the optimal selection of M, N, and b depends on the mean "fluctuation rates" in the x and y directions. Bruckstein [1] outlined an alternative approach, based on the assumption that $f(x, y)$ is a sample function of a 2-D stationary stochastic process with a known covariance function, using integral sampling and zero-order hold interpolation.

In this paper a different approach to image representation is considered. Its basis is the representation of $f(x, y)$ by a sloped plane within a pixel rather than by a horizontal plane. Three numbers are now associated with each pixel, indicating the average value, the slope in the x direction, and the slope in the y direction. Since these numbers must be quantized, a bit allocation problem arises. If b bits are allocated for each pixel, how many bits should be used to represent each of the three numbers?

The answer to this bit allocation problem is very meaningful in the evaluation of the piecewise-planar representation and constitutes the main contribution of this paper. Clearly, the number of bits that are allocated to represent the slope indicates its relative importance. Indeed, if no bits are allocated to represent the slope, this representation degenerates to standard representations.

Piecewise planar image representation itself is not a new concept. It is indeed a special case of image representation by orthogonal functions within pixel blocks and can be regarded as a compromise between standard piecewise constant image digitization and blockwise transform coding. Strobach [7] developed a useful quadtree-structured image coding scheme in which the digital image is represented within pixel-blocks by piecewise-planar functions. We regard piecewise-planar image representation as an intermediate representation that is compact with respect to standard piecewise constant representation, but still sufficiently explicit to be convenient as the input or output of various image processing algorithms.

The organization of this paper is as follows. In Section 2 optimal bit allocation is studied for the case of deterministic images. In Sections 3 and 4 the representation of images that are samples of a 2-D stationary stochastic process with a known covariance function is considered. For the specific purpose of high-quality visual display, it

[1] A part of this research was carried out while Nahum Kiryati was with the Image Science Laboratory, Institute for Communication Technology, ETH, Zurich, Switzerland. The support and hospitality of Professor Olaf Kübler and the members of the laboratory are gratefully acknowledged. This research was supported in part by the Ollendorff Center at the Department of Electrical Engineering of the Technion.

1047-3203/95 $6.00

is desirable to further interpolate the piecewise planar function. The post processing interpolation should not be confused with the intermediate representation itself. But even though the visual appearance is not necessarily an important consideration in the evaluation of an intermediate representation, it can certainly enhance our understanding. The visual effect of piecewise-planar image representation is demonstrated in Section 5.

Certain steps in the mathematical derivations that have been omitted for brevity, as well as results on the important one-dimensional case, can be found in [3]. Preliminary results were presented in [4].

2. REPRESENTATION OF DETERMINISTIC IMAGES

Let $f(x, y)$ be a function defined on a unit square $\Omega = [0, 1] \times [0, 1]$. Ω is divided into $M \times N$ rectangular cells (pixels) of size $A_x \times A_y$, where $A_x = 1/M$ and $A_y = 1/N$. Let D_{ij} (with $i = 1, \ldots, M$ and $j = 1, \ldots, N$) denote a typical pixel, and let (x_i, y_j) be the coordinates of its center.

In standard schemes $f(x, y)$ would be represented by a constant within a pixel. In this paper, the representation of $f(x, y)$ within a pixel D_{ij} is

$$\hat{f}(x, y) = C_{1_{ij}} n_1 + C_{x_{ij}} n_x (x - x_i) + C_{y_{ij}} n_y (y - y_j). \quad (1)$$

n_1, n_x, and n_y are normalization constants chosen so that the functions n_1, $n_x(x - x_i)$, and $n_y(y - y_j)$, restricted to D_{ij}, are orthonormal. In particular,

$$n_1 = (1/A_x A_y)^{1/2}$$
$$n_x = (12/A_x^3 A_y)^{1/2}$$
$$n_y = (12/A_y^3 A_x)^{1/2}.$$

$C_{1_{ij}}$, $C_{x_{ij}}$, and $C_{y_{ij}}$ are selected to minimize the average square distance between $f(x, y)$ and $\hat{f}(x, y)$, i.e., to minimize

$$\varepsilon_{ij}^2 \triangleq \frac{1}{A_x A_y} \int\int_{D_{ij}} [f(x, y) - \hat{f}(x, y)]^2 \, dx \, dy. \quad (2)$$

Clearly,

$$C_{1_{ij}} = n_1 \int\int_{D_{ij}} f(x, y) \, dx \, dy \quad (3a)$$

$$C_{x_{ij}} = n_x \int\int_{D_{ij}} (x - x_i) f(x, y) \, dx \, dy \quad (3b)$$

$$C_{y_{ij}} = n_y \int\int_{D_{ij}} (y - y_j) f(x, y) \, dx \, dy. \quad (3c)$$

Substituting the optimal $C_{1_{ij}}$, $C_{x_{ij}}$, and $C_{y_{ij}}$ in (2), it follows that

$$\varepsilon_{ij}^2 = \overline{f_{ij}^2(x, y)} - \frac{1}{A_x A_y} (C_{1_{ij}}^2 + C_{x_{ij}}^2 + C_{y_{ij}}^2),$$

where

$$\overline{f_{ij}^2(x, y)} \triangleq \frac{1}{A_x A_y} \int\int_{D_{ij}} f^2(x, y) \, dx \, dy.$$

The quantization of $C_{1_{ij}}$, $C_{x_{ij}}$, and $C_{y_{ij}}$ has been so far ignored. Taking quantization into account, the representation of $f(x, y)$ within a pixel D_{ij} is

$$\hat{f}^Q(x, y) = C_{1_{ij}}^Q n_1 + C_{x_{ij}}^Q n_x (x - x_i) + C_{y_{ij}}^Q n_y (y - y_j). \quad (4)$$

Defining

$$\varepsilon_{ij}^{Q^2} \triangleq \frac{1}{A_x A_y} \int\int_{D_{ij}} [f(x, y) - \hat{f}^Q(x, y)]^2 \, dx \, dy,$$

it is easy to show that

$$\varepsilon_{ij}^{Q^2} = \varepsilon_{ij}^2 + \frac{1}{A_x A_y} [(C_{1_{ij}} - C_{1_{ij}}^Q)^2 + (C_{x_{ij}} - C_{x_{ij}}^Q)^2 + (C_{y_{ij}} - C_{y_{ij}}^Q)^2].$$

The average square error over all pixels is

$$\varepsilon^{Q^2} \triangleq \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \varepsilon_{ij}^{Q^2}.$$

Assuming that M and N are fixed and that b, the total number of bits per pixel, is also given, we proceed to determine the optimal uniform bit allocation scheme, i.e., the numbers of bits b_1, b_x, and b_y ($b_1 + b_x + b_y = b$) that are to be used for the representation of $C_{1_{ij}}$, $C_{x_{ij}}$, and $C_{y_{ij}}$ (for every i, j) in order to minimize ε^{Q^2}. ε^{Q^2} can be written as

$$\varepsilon^{Q^2} = \varepsilon^2 + \Delta^Q,$$

where

$$\varepsilon^2 \triangleq \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \varepsilon_{ij}^2$$

is the spatial sampling component of the overall average square error, and

$$\Delta^Q \triangleq \sum_{i=1}^{M} \sum_{j=1}^{N} [(C_{1_{ij}} - C_{1_{ij}}^Q)^2 + (C_{x_{ij}} - C_{x_{ij}}^Q)^2 + (C_{y_{ij}} - C_{y_{ij}}^Q)^2]$$

is the contribution of coefficient quantization errors to the total mean square error. The allocation of bits b_1, b_x, and b_y has obviously no effect on ε^2, so only Δ^Q has to be considered in selecting their values.

Had the number of pixels MN been small with respect to 2^b, it would have been in principle possible to assign an exact code to each of the triplets $(C_{1_{ij}}, C_{x_{ij}}, C_{y_{ij}})$ and avoid quantization errors altogether. This is however not the usual case, MN typically being much larger than 2^b (e.g., $512 \times 512 \gg 256$). It is then possible to quantize $C_{1_{ij}}$, $C_{x_{ij}}$, and $C_{y_{ij}}$ using the (optimal) Max–Lloyd quantizer [5], regarding each of the coefficients to be quantized as a sample from the distribution of $\{C_{1_{ij}}\}$, $\{C_{x_{ij}}\}$, or $\{C_{y_{ij}}\}$ values in the image. (Note that alternative scalar quantizers could have been selected. For higher coding efficiency at a price of less explicit representation, vector quantization could also be applied).

Assume that

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{1_{ij}} = 0$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{x_{ij}} = 0$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{y_{ij}} = 0.$$

These assumptions are made without loss of generality: simply assume that the plane that corresponds to the average value and slope was already subtracted from the image.

Following the notation of [2], Δ^Q can be expressed for the case of Max–Lloyd quantization as

$$\Delta^Q \cong \frac{K_{Q_1}(b_1)}{2^{2b_1}} \sum_{i=1}^{M}\sum_{j=1}^{N} C_{1_{ij}}^2 + \frac{K_{Q_x}(b_x)}{2^{2b_x}} \sum_{i=1}^{M}\sum_{j=1}^{N} C_{x_{ij}}^2 + \frac{K_{Q_y}(b_y)}{2^{2b_y}} \sum_{i=1}^{M}\sum_{j=1}^{N} C_{y_{ij}}^2, \tag{5}$$

where $K_{Q_1}(b_1)$, $K_{Q_x}(b_x)$, and $K_{Q_y}(b_y)$ are typically slowly varying functions whose exact forms depend on the particular distributions of the values of $C_{1_{ij}}$, $C_{x_{ij}}$, and $C_{y_{ij}}$ respectively.

Using (3a) it is clear that

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{1_{ij}}^2 = \sum_{i=1}^{M}\sum_{j=1}^{N} \left[\overline{f_{ij}(x, y)}\right]^2 A_x A_y,$$

where

$$\overline{f_{ij}(x, y)} \triangleq \frac{1}{A_x A_y} \int\int_{D_{ij}} f(x, y)\, dx\, dy.$$

Assuming that M and N are large, i.e., that A_x and A_y are small,

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{1_{ij}}^2 \approx \int\int_{\Omega} f^2(x, y)\, dx\, dy. \tag{6}$$

Using (3b) and expanding $f(x, y)$ to a Taylor series in the variable x around x_i, it is found that

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{x_{ij}}^2 \approx \frac{A_x^2}{12} \int\int_{\Omega} \left[\frac{\partial f(x, y)}{\partial x}\right]^2 dx\, dy \tag{7a}$$

and in a similar way

$$\sum_{i=1}^{M}\sum_{j=1}^{N} C_{y_{ij}}^2 \approx \frac{A_y^2}{12} \int\int_{\Omega} \left[\frac{\partial f(x, y)}{\partial y}\right]^2 dx\, dy. \tag{7b}$$

Substituting (6), (7a), and (7b) in (5) we obtain

$$\Delta^Q \cong \frac{K_{Q_1}(b_1)}{2^{2b_1}} \int\int_{\Omega} f^2(x, y)\, dx\, dy + \frac{K_{Q_x}(b_x)}{2^{2b_x}} \cdot \frac{A_x^2}{12} \cdot \int\int_{\Omega} \left[\frac{\partial f(x, y)}{\partial x}\right]^2 dx\, dy + \frac{K_{Q_y}(b_y)}{2^{2b_y}} \cdot \frac{A_y^2}{12} \cdot \int\int_{\Omega} \left[\frac{\partial f(x, y)}{\partial y}\right]^2 dx\, dy.$$

Optimal bit allocation implies

$$\frac{K_{Q_1}(b_1)}{2^{2b_1}} \int\int_{\Omega} f^2(x, y)\, dx\, dy = \frac{K_{Q_x}(b_x)}{2^{2b_x}} \cdot \frac{A_x^2}{12} \cdot \int\int_{\Omega} \left[\frac{\partial f(x, y)}{\partial x}\right]^2 dx\, dy = \frac{K_{Q_y}(b_y)}{2^{2b_y}} \cdot \frac{A_y^2}{12} \cdot \int\int_{\Omega} \left[\frac{\partial f(x, y)}{\partial y}\right]^2 dx\, dy,$$

therefore the optimal allocation of bits satisfies

$$b_1 - b_x = \frac{1}{2}\log_2\left\{\frac{K_{Q_1}(b_1)}{K_{Q_x}(b_x)} \cdot \frac{12}{A_x^2} \cdot \frac{\int\int_{\Omega} f^2(x, y)\, dx\, dy}{\int\int_{\Omega} [(\partial f(x, y))/(\partial x)]^2\, dx\, dy}\right\} \tag{8a}$$

$$b_1 - b_y = \frac{1}{2}\log_2\left\{\frac{K_{Q_1}(b_1)}{K_{Q_y}(b_y)} \cdot \frac{12}{A_y^2} \cdot \frac{\int\int_{\Omega} f^2(x, y)\, dx\, dy}{\int\int_{\Omega} [(\partial f(x, y))/(\partial y)]^2\, dx\, dy}\right\}. \tag{8b}$$

Since $K_{Q_1}(b_1)$, $K_{Q_x}(b_x)$, and $K_{Q_y}(b_y)$ are usually slowly varying functions, (8) is in fact an explicit solution. In actual applications the optimal values of b_1, b_x, and b_y must be replaced by feasible natural numbers.

Consider, for example, the function

$$f(x, y) = \sin(\omega_x x)$$

and assume that ω_x is large. Then

$$\int\int_\Omega f^2(x, y)\, dx\, dy \cong \frac{1}{2}$$

$$\int\int_\Omega \left[\frac{\partial f(x, y)}{\partial x}\right]^2 dx\, dy \cong \frac{1}{2}\omega_x^2$$

$$\int\int_\Omega \left[\frac{\partial f(x, y)}{\partial y}\right]^2 dx\, dy = 0.$$

Thus,

$$b_1 - b_x \cong \frac{1}{2}\log_2\left[\frac{K_{Q_1}(b_1)}{K_{Q_x}(b_x)} \cdot \frac{12}{A_x^2} \cdot \frac{1}{\omega_x^2}\right]$$

and

$$b_1 - b_y \to \infty,$$

i.e., b_y should be set to zero. This is a pleasing result, since $f(x, y)$ is constant along lines parallel to the y-axis.

Using the approximation[2]

$$\log_2 \frac{K_{Q_1}(b_1)}{K_{Q_x}(b_x)} \approx 0,$$

it follows that

$$b_1 - b_x \approx \frac{1}{2}\log_2 \frac{12}{A_x^2\omega_x^2}.$$

It is apparent that the relative importance of the slope data (as implied by b_x) increases as the pixel size grows (in the x direction), as long as $A_x\omega_x$ is not too large.

3. REPRESENTATION OF REALIZATIONS OF 2-D STOCHASTIC PROCESSES

In this section it is assumed that the image function is a realization of a 2-D zero-mean wide-sense stationary random process $\{f_\omega(x, y)\}$, with a given autocorrelation function $R(\tau_x, \tau_y)$, and that the optimal uniform bit allocation scheme that minimizes the mean square error has to be determined. In particular, it is again assumed that the spatial sampling parameters M and N, and the total number of bits per pixel b, are fixed. b_1, b_x, and b_y must be determined such that

$$E\varepsilon^{Q^2} = E\int\int_\Omega [f_\omega(x, y) - \hat{f}^Q(x, y)]^2\, dx\, dy,$$

i.e., the expected representation error, would be minimized.

The mean-squared error over the individual pixels has the same expectation for each pixel. Thus

$$E\varepsilon^{Q^2} = E\varepsilon_D^{Q^2} = E\frac{1}{A_xA_y}\int\int_D \{[f_\omega(x, y) - \hat{f}(x, y)] + [\hat{f}(x, y) - \hat{f}^Q(x, y)]\}^2\, dx\, dy,$$

where $\hat{f}(x, y)$ and $\hat{f}^Q(x, y)$ are respectively the optimal representation of $f_\omega(x, y)$ in the form of (1) within the pixel D, and its quantized version in the form of (4). The assumption that $f_\omega(x, y)$ is zero mean and the optimal selection of C_1, C_x, and C_y by (3) imply that $EC_1 = EC_x = EC_y = 0$.

The main goal of this analysis is to determine the best allocation of the b available bits per pixel between b_1, b_x, and b_y, the numbers of bits allocated to C_1, C_x, and C_y. Assuming Max–Lloyd (optimal) quantization of C_1, C_x, and C_y, and using again the notation of [2], it is easy to show [3] that

$$E\varepsilon^{Q^2} = R(0, 0) - \frac{1}{A_xA_y}\left[\left(1 - \frac{K_{Q_1}}{2^{2b_1}}\right)EC_1^2 + \left(1 - \frac{K_{Q_x}}{2^{2b_x}}\right)EC_x^2 + \left(1 - \frac{K_{Q_y}}{2^{2b_y}}\right)EC_y^2\right],$$

where K_{Q_1}, K_{Q_x}, and K_{Q_y} are typically slowly varying functions of b_1, b_x, and b_y, respectively, that can be often regarded as constants.

It can be further shown [3] that

$$E\varepsilon^{Q^2} = R(0, 0) - \frac{4}{A_xA_y}\int_0^{A_x}\int_0^{A_y}\left(1 - \frac{x}{A_x}\right)\left(1 - \frac{y}{A_y}\right)R(x, y) \left\{\left(1 - \frac{K_{Q_1}}{2^{2b_1}}\right) + \left(1 - \frac{K_{Q_x}}{2^{2b_x}}\right)\left(1 - 2\frac{x}{A_x} - 2\frac{x^2}{A_x^2}\right) + \left(1 - \frac{K_{Q_y}}{2^{2b_y}}\right)\left(1 - 2\frac{y}{A_y} - 2\frac{y^2}{A_y^2}\right)\right\} dy\, dx. \quad (9)$$

Equation (9) is probably as far as one can go without specifying the autocorrelation function $R(\tau_x, \tau_y)$ of the random process $\{f_\omega(x, y)\}$.

[2] For a justification of a similar approximation see [3].

4. (ALMOST) EXPLICIT SOLUTION FOR MARKOV PROCESSES

Suppose that the process $\{f_\omega(x, y)\}$ is a separable, 2-D Markov process, i.e., that its autocorrelation function is

$$R(\tau_x, \tau_y) = e^{-\alpha_x|\tau_x|}e^{-\alpha_y|\tau_y|}. \tag{10}$$

Substituting (10) in (9) and assuming that A_x and A_y are small enough so that $\alpha_x A_x$ and $\alpha_y A_y$ are small too, it can be shown [3] that

$$\begin{aligned} E\varepsilon_Q^2 \cong 1 &- \left(1 - \frac{1}{3}\alpha_x A_x\right)\left(1 - \frac{1}{3}\alpha_y A_y\right)\left(1 - \frac{K_{Q_1}}{2^{2b_1}}\right) \\ &- \frac{1}{5}\alpha_x A_x\left(1 - \frac{1}{3}\alpha_y A_y\right)\left(1 - \frac{K_{Q_x}}{2^{2b_x}}\right) \\ &- \frac{1}{5}\alpha_y A_y\left(1 - \frac{1}{3}\alpha_x A_x\right)\left(1 - \frac{K_{Q_y}}{2^{2b_y}}\right) \end{aligned}$$

and that optimal bit allocation satisfies

$$b_1 - b_x \cong \frac{1}{2}\log_2\left(\frac{K_{Q_1}}{K_{Q_x}} \cdot \frac{1 - (1/3)\alpha_x A_x}{(1/5)\alpha_x A_x}\right) \tag{11}$$

and

$$b_1 - b_y \cong \frac{1}{2}\log_2\left(\frac{K_{Q_1}}{K_{Q_y}} \cdot \frac{1 - (1/3)\alpha_y A_y}{(1/5)\alpha_y A_y}\right). \tag{12}$$

FIG. 1. The 512 × 512 spatial resolution of this image is significantly higher than in any of the next figures. This image is thus presented as a reasonable approximation to the original continuous picture.

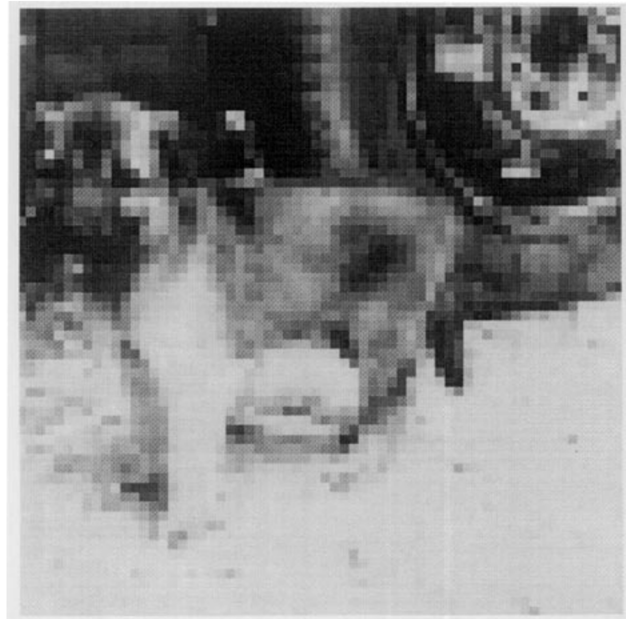

FIG. 2. A standard representation using 64 × 64 pixels and 8 bits per pixel for grey level quantization. The grey level is constant within each pixel. This image is the reference to which Figs. 3 and 4 should be compared.

Equations (11) and (12), taken together with the constraint $b_1 + b_x + b_y = b$, give the (nearly) optimal values of b_1, b_x, and b_y under the above-mentioned assumptions.

5. THE VISUAL EFFECT OF PIECEWISE-PLANAR IMAGE REPRESENTATION

The visual demonstration begins with the 512 × 512 picture shown in Fig. 1. Lower resolution image representations are used in the sequel, so Fig. 1 can be regarded as a reasonable approximation of the original continuous image. Figure 2 corresponds to standard image digitization, with a constant grey level within each pixel. The spatial resolution is 64 × 64 and 8 bits per pixel are allocated to grey level quantization. This is the reference to which the piecewise-planar image representation should be compared. The crude spatial resolution enhances the visibility of the artifacts induced by the digitization and representation processes.

Piecewise-planar representation, at 64 × 64 spatial resolution but without quantization of the coefficients within the pixels, is shown in Fig. 3. Figure 3 is obviously better than Fig. 2. For honest comparison with the standard method, 8 bits should be allocated to each of the 64 × 64 pixels. The optimal bit allocation was found to be $b_1 = 5$, $b_x = 1$, and $b_y = 2$. The resulting image is shown in Fig. 4 and is clearly better than Fig. 2, but naturally is not as good as Fig. 3. These images demonstrate the representation itself, without any interpolation.

FIG. 3. Piecewise planar image representation at a spatial resolution of 64 × 64, but without quantization of the three coefficients within each pixel. This image demonstrates the potential quality of piecewise planar representation even with quite low spatial resolution.

FIG. 5. A standard representation using 128 × 128 pixels and 8 per pixel for grey level quantization. The grey level is constant within each pixel. This image is the reference to which Figs. 6 and 7 should be compared.

Figures 5–7 repeat the demonstration at a spatial resolution of 128 × 128. Figure 5 corresponds to standard, piecewise constant image representation with 8 bits per pixel for grey level quantization. Figure 6 demonstrates piecewise-planar representation without quantization of the coefficients, and Fig. 7 shows piecewise-planar representation with 8 bits allocated per pixel. The optimal allocation is again $b_I = 5$, $b_x = 1$, and $b_y = 2$. It is seen that

FIG. 4. *Piecewise planar image representation at a spatial resolution* of 64 × 64 with 8 bits per pixel. The average grey level is represented using 5 bits, the slope in the x direction by 1 bit and the slope in the y direction by 2 bits.

FIG. 6. Piecewise planar image representation at a spatial resolution of 128 × 128, but *without quantization of the three coefficients* within each pixel.

FIG. 7. Piecewise planar image representation at a spatial resolution of 128 × 128 with 8 bits per pixel. The average grey level is represented using 5 bits, the slope in the x direction by 1 bit, and the slope in the y direction by 2 bits.

with the same spatial resolution and total number of bits per pixel, piecewise-planar image representation has better visual appearance than standard digitization.

REFERENCES

1. A. M. Bruckstein, On optimal image digitization, *IEEE Trans. Acoust. Speech Signal Process.* **ASSP-35,** Apr. 1987, 553–555.
2. J. J. Y. Huang and P. M. Schultheiss, Block quantization of correlated Gaussian random variables, *IEEE Trans. Comm. Systems* **CS-11,** Sept. 1963, 289–296.
3. N. Kiryati and A. M. Bruckstein, *On Piecewise Planar Representation of Images,* Tech. Rep. BIWI-TR-123, Institute for Communication Technology, ETH Zurich, July 1991.
4. N. Kiryati and A. M. Bruckstein, On piecewise planar representation of images, in *Proceedings of the 11th ICPR, The Hague, The Netherlands, 31 Aug.–3 Sept. 1992,* Vol. C, pp. 451–454.
5. J. Max, Quantizing for minimum distortion, *IRE Trans. Inform. Theory* **IT-6,** Mar. 1960, 7–12.
6. L. Nielsen, K. J. Aström, and E. I. Jury, Optimal digitization of 2-D images, *IEEE Trans. Acoust. Speech Signal Process.* **ASSP-32,** Dec. 1984, 1247–1249.
7. P. Strobach, Quadtree-structured recursive plane decomposition coding of images, *IEEE Trans. Acoust. Speech Signal Process.* **ASSP-39,** June 1991, 1380–1397.

NAHUM KIRYATI was born in Haifa, Israel, in 1958. He received the B.Sc. degree in electrical engineering and the Post-B.A. degree in the Humanities from Tel Aviv University, Israel, in 1980 and 1986, respectively. He received the M.Sc. degree in electrical engineering in 1988 and the D.Sc. degree in 1991, both from the Technion, Israel Institute of Technology, Haifa, Israel. From April 1991 to February 1992 he was with the Image Science Laboratory, Institute for Communication Technology, Swiss Federal Institute of Technology, Zurich, Switzerland. He is now with the Department of Electrical Engineering at the Technion, Haifa. His research interests are in image analysis and computer vision.

ALFRED M. BRUCKSTEIN was born in Transylvania, Romania, on January 24, 1954. He received the B.Sc. and M.Sc. degrees in electrical engineering from the Technion, Israel Institute of Technology, in 1977 and 1980, respectively, and the Ph.D. degree in electrical engineering from Stanford University, Stanford, California, in 1984. Since October 1984 he has been with the Technion, Haifa, Israel. He is a frequent visitor at AT&T's Bell Laboratories, Murray Hill, New Jersey. His present research interests are in computer vision, pattern recognition, image processing, and computer graphics. He has also done work in estimation theory, signal processing, algorithmic aspects of inverse scattering, point processes and mathematical models in neurophysiology. Professor Bruckstein is a member of SIAM, MAA, and AMS.

AMNON JONAS was born in Jerusalem, Israel, on September 17, 1966. He received the B.Sc. degree in electrical engineering in 1988 from the Technion—Israel Institute of Technology, Haifa, Israel. He is currently studying toward the M.Sc. degree in electrical engineering at the Technion and is also a teaching assistant there. His research interests are in digital image and signal processing, digital geometry, and three-dimensional shape analysis.