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Efficient cooperative search of smart targets using UAV Swarms[1]

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SUMMARY
This work examines the *Cooperative Hunters* problem, where a swarm of unmanned air vehicles (UAVs) is used for searching one or more "evading targets," which are moving in a predefined area while trying to avoid a detection by the swarm. By arranging themselves into efficient geometric flight configurations, the UAVs optimize their integrated sensing capabilities, enabling the search of a maximal territory.

KEYWORDS: Swarm Algorithm, Cooperative Search, Shape Factor.

1. Introduction

In the world of living creatures, "simple minded" animals such as ants or birds cooperate to achieve common goals with surprising performance. It seems that these animals are "programmed" to interact locally in such a way that the desired global behavior is likely to emerge even if some individuals of the colony die or fail to carry out their task for some other reasons. It is suggested to consider a similar approach to coordinate a group of robots without a central supervisor, by using only local interactions between the robots. When this decentralized approach is used, much of the communication overhead (characteristic to centralized systems) is saved, the hardware of the robots can be fairly simple, and better modularity is achieved. A properly designed system should achieve reliability through redundancy. In addition, the scalability that is obtained by using a decentralized design often results in a robotics system whose performance can successfully compete with a centralized approach. Furthermore, the analysis of such systems are usually much easier.

In recent years significant research efforts have been invested in design and simulation of multi-agent robotics and intelligent swarms systems—see, e.g., refs. [2–4] or [5–7] for biology inspired designs (behavior-based control models, flocking and dispersing models, and predator–prey approaches, respectively), refs. [20–23] for economics applications, and refs. [24, 35] for physics-inspired approaches).

Tasks that have been of particular interest to researchers in recent years include synergetic mission planning,[8] fault tolerance,[9] swarm control,[10] human design of mission plans,[11] role assignment,[12] multi-robot path planning,[13] traffic control,[14] formation generation,[15] formation keeping,[16] exploration and mapping,[17] cleaning[18] and dynamic cleaning[19] and target tracking.[26]

Unfortunately, the mathematical geometrical theory of such multi-agent systems is far from being satisfactory, as pointed out in ref. [25] and many other papers.

One of the most interesting challenges for a robotics swarm system is the design and analysis of a multi-robotics system for searching areas (either known or unknown),[27–31,34] or see ref. [37] for a survey of search and evasion strategies. Interesting works to mention in this scope are those of refs. [32, 33], where a swarm of ant-like robots is used for repeatedly covering an unknown area, using a real-time search method called *node counting*. By using this method, the robots are shown to be able to efficiently perform such a coverage mission, and analytic bound for the coverage time are discussed.

While in most works the targets of the search mission were assumed to be idle, recent works considered dynamic targets, meaning targets, which after being detected by the searching robots, respond by performing various evasive maneuvers intended to prevent their interception. In this context it is interesting to mention the roots of this field, dating back to World War II.[39,40] The first planar search problem considered is the patrol of a corridor between parallel borders separated by width W. This problem was solved in ref. [38] in order to determine optimal patrol strategies for aircraft searching for ships in a channel.

A similar problem was presented in ref. [1], where a system consisting of a swarm of UAVs (unmanned air vehicles) was designed to search for one or more such "smart targets" (representing for example enemy units, or alternatively a lost friendly unit which should be a found and rescued). In this problem (presented in Section 2, the objective of the UAVs is to find the targets in the shortest time possible. While the swarm comprises relatively simple UAVs, lacking prior knowledge of the initial positions of the targets, the targets are equipped with strong sensors, capable of telling the locations of the UAVs from very long distances. The search strategy

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suggested in ref. [1] defines *flying patterns* which the UAVs follow, designed for scanning the (rectangular) area in such a way that the targets cannot re-enter sub-areas which were already scanned by the swarm, without being detected (a summary of this solution is presented in Section 3).

This work suggests an improved geometric approach for designing and analyzing such a "cooperative hunters" robotics system. This approach is presented in Section 4, demonstrating a design of a system, which uses improved "flying patterns" for achieving an efficient hunt. The efficiency of the new geometric flying configuration is analyzed, and compared both to techniques discussed in previous works and to the optimal solution for the problem.

2. The Cooperative Hunters Problem

As described above, the Cooperative Hunters problem discusses a swarm of UAVs utilized for searching and intercepting a set of evading targets. Following are more details concerning this model, as presented in ref. [1]. Note that in the model described below several "real life" properties (such as sensors' errors or search regions which have curved topographies) were intentionally omitted, in order to provide a clear view of the geometric analysis system.

2.1. The targets

The targets are able to move at a (known) maximum speed of V_{target}. Aside from knowing the targets' maximal speed, the UAVs possess no additional information regarding each target's actual speed or course. Each target is free to adjust its course and speed and is, in fact, assumed to be capable of intelligent evasion. Each target may spot the searcher UAVs from a distance (far beyond the searchers' detection range) and subsequently maneuver in an attempt to evade detection. This work focuses on the detection task only, leaving the actual tracking and handling of the targets once discovered out of the scope of this paper.

2.2. UAVs and sensors

To carry out the search, a group of identically configured UAVs is given; each flies at a constant velocity of V_{UAV}. We assume that each aircraft can detect targets (using its sensors of range D) according to a definite range law of detection—specifically, the aircraft will always detect a target that is placed within a radial distance of $\frac{D}{2}$ from it, and will never detect targets which are located beyond this range.[2] The UAVs are equipped with Global Positioning System (GPS) receivers so that they can accurately ascertain the coordinates of their location at any time. Additionally, each UAV is aware of the geographic boundaries of the search region.

2.3. The search region

The search region is assumed to be known in advance, that is the targets are known to be confined to a specific finite planar region. This assumption originated from the fact that a vehicle convoy might, for example, be surrounded by mountains, water and other features that restrict its location to a large (but limited) area. Similarly, a small boat patrolling crowded shipping lanes might have a fuel supply that restricts how far from the coastline it might proceed, or alternatively a small yacht lost as a lake which should be found and rescued can be assumed to stay in the water. Simply put, each target's location is confined at all times to a rectangular region of width X, length Y and area $A = X \cdot Y$. Without limitation of generality, the label X is assigned to the shorter side of the region (i.e., $X \leq Y$) and note that X is usually considerably larger than the sensor's detection diameter D. Apart from being confined within the rectangular boundaries of the search region, no other information concerning the targets' locations is available (as such, we assume a target's location is uniformly distributed over this region).

2.4. The goal

The objective of this work, as declared in ref. [1], is to develop an efficient search methodology for employing a UAV swarm in order to efficiently locate intelligent and evading targets within the search region. The search algorithm should attempt to maximize the probability of targets' detection and minimize the expected search time while minimizing the number of UAVs required. The algorithm should adapt in the face of UAV failures by reconfiguring the UAVs to optimally continue the mission with the surviving assets. The execution of the algorithm (including any adaptations necessitated by the loss of UAVs) should be accomplished with a minimum amount of control information to be passed between UAVs.

In the spirit of designing a system which uses as simple agents as possible, we aspire that the agents will have as little communication capabilities as possible. With respect to the taxonomy of multi-agents discussed in ref. [36], we would be interested in using agents of the types *COM-NONE* or if necessary of type *COM-NEAR* with respect to their communication distances, and of types *BAND-MOTION*, *BAND-LOW* or even *BAND-NONE* (if possible) with respect to their communication bandwidth.

3. Previous Results

A search algorithm for the UAVs was presented in ref. [1], designed to both limit the amount of information that must be exchanged between UAVs and to use simple algorithms to modify the search in the event of a UAV loss. This solution used a small group of pre-defined swarm flying patterns. The flying patterns guarantee a successful completion of the mission, while their limited number allows an efficient and automatic reconfiguration of the UAVs array, in the case of a UAV malfunction. The initial number of UAVs in the pattern is decided (by ground commanders) prior to launch, and each UAV is given the pattern, its position in the pattern and the dictionary of allowable alternative patterns.

To achieve high searching efficiency, the UAVs form and maintain a configuration of a straight line, as can be seen in Fig. 1. The line formation then moves and scans a portion of the rectangle (of size S) by moving *south*. Upon reaching the boundaries of the search region, the formation travels a constant distance *eastwards*, and scans another portion of the search region, by going *north*. Note that both scanned

[2] As in reality, the sensors have finite detection time, the value of D used in the model can be slightly smaller than the actual sensors' range, in order to generate positive overlap between a pair of UAVs moving in parallel (see more details in Section 4).

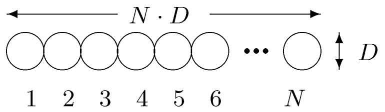

Fig. 1. A line formation of the UAVs, demonstrating a static detection area of $N \cdot D$.

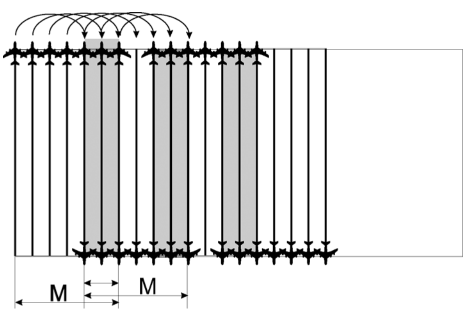

Fig. 2. An illustration of the algorithm proposed by Vincent and Rubin.[1]

areas overlap each other to prevent targets from re-entering a scanned area. The overlap region between both scanned areas is of size B. This process continues repeatedly, until the eastern boundaries of the search area are met, at which time all targets are guaranteed to be detected. A visual demonstration of the above is presented in Fig. 2.

The following result appears in ref. [1] and demonstrates a lower bound over the number of UAVs required to ensure a successful completion of the search mission:

Lemma 1. Assuming we have N identically configured *UAVs*, moving at velocity V_{UAV}, arranged in a line formation, such that their effective sweep width is $N \cdot D$, searching for targets which are capable of moving at velocity V_{target}. Then, the minimal number of UAVs required to ensure a forward-moving search with a detection probability equals to one is given by:

$$N_{min} = \min\left\{\left\lfloor \frac{2X \cdot V_{target}}{V_{UAV} \cdot D} \right\rfloor + 1, \left\lceil \frac{Y}{D} \right\rceil\right\}.$$

4. Improved Geometrical Flying Patterns

Let N, D, M, Y and X denote the number of UAVs, their sensors' detection diameter, the line formation's scan width (note that $M = N \cdot D$), the length of the rectangular region and the width of the rectangular region, respectively. We now show an algorithm that can guarantee a successful detection of the targets with half the number of UAVs required for the algorithm of ref. [1].

4.1. Description of the algorithm

Similar to the algorithm of ref. [1] we use a repeated scanning formation (north–south and vice versa), sweeping by a line formation of UAVs (as illustrated in Fig. 3).

Before each sweeping pass, the UAVs are located at either the northern or southern boundary of the search region, as a line formation. In Fig. 3 two sweeping passes are demonstrated. Without loss of generality let us assume that the first pass starts at time $t = 0$. Before the first and after

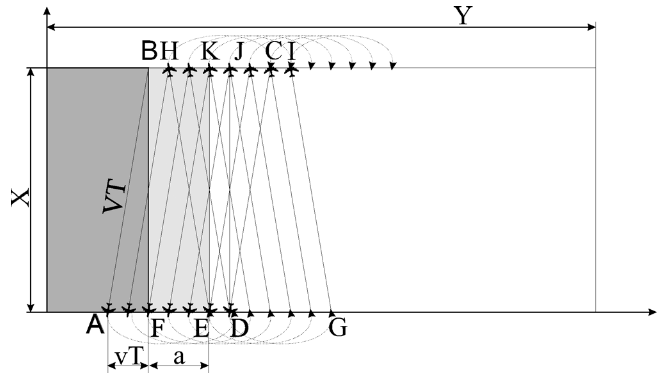

Fig. 3. An illustration of the improved searching algorithm.

the second pass the planes are at the northern boundary of the search region (a light-gray rectangle shows the addition to the "clean" area after the two passes).

The line formation first moves from the segment BC to AD. For the sake of simplicity, we assume the region west to the segment BF (appeared as dark gray in Fig. 3) to already be clean at time $t = 0$.

The points A and D towards which the UAVs formation heads are selected in such a way so that $\frac{|AB|}{V_{\text{UAV}}} = \frac{|AF|}{V_{\text{target}}}$ (thus preventing the targets from re-entering the already cleaned region west of the line AB). Let us assume that $V_{\text{UAV}} > V_{\text{target}}$ and denote $T = \frac{|AB|}{V_{\text{UAV}}} = \frac{|AF|}{V_{\text{target}}}$.

Lemma 2. When the formation reaches AD, the southern side of the rectangle at time $t = T$, the region to the west of line DJ is clean.

Proof. We partition the region to the west of line DJ to two parts and prove the lemma for each of them separately:

1. No target can move to the west of line AB. Suppose a target crosses the line at some point $Z \in [AB]$. We assume that the target moved directly westwards, in order to reach a maximal distance from the UAVs (any other can be treated as a westwards movement with a smaller V_{target}). Then, by the choice of A, if at time $t = 0$ the target was in $Q \in [BF]$, where Q is such that ZQ is parallel to axis X and the UAV at B, both the UAV and the target would reach Z simultaneously and the target will not cross AB undetected.
2. A target cannot move into $ABJD$. Indeed if it waits at line CD until the UAVs formation passes, and then heads westwards, by the choice of point A the target will reach line JD at the same time as the formation reaches the southern side of the area, and hence will be detected. □

When the formation reaches the segment AD, it shifts eastwards to occupy segment EG, in such a way that the area to the west of EK is guaranteed to be clean. Hence E is chosen such that $\frac{|AE|}{V_{\text{UAV}}} = \frac{|ED|}{V_{\text{target}}}$. This completes the description of one pass of the algorithm (as from EG the formation moves northwards to HI, then eastwards, etc.).

4.2. Analysis of the algorithm

In this section we shall analyze the behavior of the algorithm for different values of V_{UAV}, V_{target} and X. First, we find the minimal ratio $\frac{V_{\text{UAV}}}{V_{\text{target}}}$, given X, for which a successful completion of the mission is guaranteed.

Lemma 3. If $\frac{V_{UAV}}{V_{target}} \geq \frac{X}{M}$ then the UAVs formation is able to move eastwards while maintaining the area west to it clean.

Proof. In order for the UAVs formation to be able to move eastwards while maintaining a clean area behind, each additional algorithm's pass must result in increasing the clean area to the west of the UAVs formation by some region of length $a > 0$. This can be maintained as long as the time it takes the formation to move from AD to EG is smaller than the time it takes a potential target to move from DJ to KE.

Hence, it must hold that $\frac{|AF|+a}{V_{\text{UAV}}} > \frac{|FD|-a}{V_{\text{target}}}$, when A is chosen such that $\frac{|BA|}{V_{\text{UAV}}} = \frac{|FA|}{V_{\text{target}}}$ (namely, the selected flying angle of the UAVs formation). That is, for the maximal possible propagation a we have

$$\begin{cases} X^2 + (V_{\text{target}}T)^2 = (V_{\text{UAV}}T)^2 \\ \dfrac{V_{\text{target}}T + a}{V_{\text{UAV}}} = \dfrac{M - V_{\text{target}}T - a}{V_{\text{target}}} \end{cases}, \quad (1)$$

which can also be written in the following way:

$$\begin{cases} T^2 = \dfrac{X^2}{V_{\text{UAV}}^2 - V_{\text{target}}^2} \\ a(V_{\text{target}} + V_{\text{UAV}}) = MV_{\text{UAV}} - V_{\text{target}}V_{\text{UAV}}T - V_{\text{target}}^2 T. \end{cases} \quad (2)$$

As we are interested in assuring that $a > 0$, and as both $V_{\text{UAV}} > 0$ and $V_{\text{target}} > 0$, we know that also $a(V_{\text{target}} + V_{\text{UAV}}) > 0$, and as a result, using Eq. 2 we know that

$$MV_{\text{UAV}} - V_{\text{target}}V_{\text{UAV}}T - V_{\text{target}}^2 T > 0,$$

meaning that

$$MV_{\text{UAV}} > (V_{\text{target}} + V_{\text{UAV}})V_{\text{target}}T.$$

As both sides are positive, rewriting Eq. 2 we get

$$\begin{cases} T^2 = \dfrac{X^2}{V_{\text{UAV}}^2 - V_{\text{target}}^2} \\ M^2 V_{\text{UAV}}^2 > (V_{\text{UAV}} + V_{\text{target}})^2 V_{\text{target}}^2 T^2. \end{cases} \quad (3)$$

Assigning the value of T^2 into the second part of Eq. 3 we get

$$M^2 V_{\text{UAV}}^2 > (V_{\text{UAV}} + V_{\text{target}})^2 V_{\text{target}}^2 \frac{X^2}{V_{\text{UAV}}^2 - V_{\text{target}}^2},$$

which we shall rewrite as

$$\frac{V_{\text{UAV}} - V_{\text{target}}}{V_{\text{UAV}} + V_{\text{target}}} \cdot \frac{V_{\text{UAV}}^2}{V_{\text{target}}^2} > \frac{X^2}{M^2}. \quad (4)$$

Then, by denoting $r = \frac{V_{\text{UAV}}}{V_{\text{target}}}$ Eq. 4 can be written as

$$\frac{r^2(r-1)}{r+1} > \frac{X^2}{M^2} \quad (5)$$

and, finally,

$$r^3 - r^2 - \frac{X^2}{M^2}r - \frac{X^2}{M^2} > 0. \quad (6)$$

A non-tight bound on r to satisfy Eq. 6 is

$$r > \frac{X}{M} + 1. \quad (7)$$

□

Thus, given a sensor's detection diameter D, using the velocities ratio result of Eq. 7, the minimal number of UAVs required to guarantee a successful completion of the mission can be obtained:

$$N_{\min} = \min\left\{ \left\lfloor \frac{X \cdot V_{\text{target}}}{V_{\text{UAV}} \cdot D} \right\rfloor + 1, \left\lceil \frac{Y}{D} \right\rceil \right\} \quad (8)$$

This can now be compared to the result of ref. [1], presented in Lemma 1, as follow:

Theorem 1. *Let $N_{min_{Vincent}}$ and $N_{min_{Hunter}}$ denote the minimal number of UAVs required to guarantee a successful completion of the mission using the algorithm presented in ref.* [1] *and in Section* 4.1, *respectively. Then, excluding the trivial case (of $N_{min_{Vincent}} = N_{min_{Hunter}} = \lceil \frac{Y}{D} \rceil$)*

$$N_{min_{Hunter}} \leq \left\lceil \frac{1}{2} N_{min_{Vincent}} \right\rceil + 1.$$

4.3. Lower bound on the number of UAVs—Optimality proof

As shown in the previous section that if $r > \frac{X}{M} + 1$, mission completion of a UAV's formation using the proposed algorithm can be guaranteed, we would now like to prove the following theorem:

Theorem 2. *If $r < \frac{X}{M}$, the UAVs will not be able to complete their mission, regardless of the algorithm they employ.*

Proof. Given a cooperative hunting algorithm, denote by $C(t)$ and $S(t)$ the convex hull of the region guaranteed to be clean of targets at time t, and its area, respectively. We shall now examine the behavior of $\frac{\partial S}{\partial t}$. Assuming that $r < \frac{X}{M}$, we show that if t is such that $S(t) = \frac{YX}{2}$ then $\frac{\partial S}{\partial t} < 0$, proving that the algorithm will not be able to complete its mission, once a certain targets-free region has been secured. Denote with $P(t)$ the length of the circumference of $C(t)$ that is not part of the rectangle's boundary. As the effective scanning region of the UAVs equals M, then during Δt time the UAVs can increase the area of the targets-free zone by at most $\Delta t M \cdot V_{\text{UAV}}$. As the targets can re-enter the targets-free zone from its boundaries (but not from the exterior of the bounding rectangle), they can decrease its area proportionally to its perimeter $P(t)$. Therefore, we can see that

$$S(t+\Delta t) - S(t) < \Delta t\, M \cdot V_{\text{UAV}} - \Delta t\, P(t) \cdot V_{\text{target}}. \quad (9)$$

By dividing both sides by Δt we get:

$$\frac{\partial S}{\partial t} \le V_{\text{target}} \cdot (r \cdot M - P(t)),$$

and by using the assumption that $\frac{X}{M} > r$ we can see that:

$$\frac{\partial S}{\partial t} < V_{\text{target}} \cdot (X - P(t)). \quad (10)$$

We now need to show that $(S(t) = \frac{YX}{2}) \Rightarrow (X < P(t))$, which in turn is shown in Lemma 4. □

Lemma 4. For a convex region $C(t)$ contained entirely in a rectangle of dimensions X and Y (X being the short side), of area $S(t) = \frac{XY}{2}$, with sub-perimeter not touching the rectangle's boundaries $P(t)$, it holds that $P(t) \ge X$.

Proof. We will divide the proof to several complementary parts, with respect to the number of sides of the search rectangle, which the convex region C(t) touches.

- $C(t)$ does not touch any of the sides of the search region—As the shape of a given area which has the minimal perimeter is a sphere, it is clear that $P(t)$ is greater or equal to the perimeter of a sphere of an area $S(t)$, meaning that: $P(t) > \sqrt{2\pi XY}$. As $X \le Y$ and since $\sqrt{2\pi} > 1$, we see that $P(t) > X$.
- $C(t)$ touches a single side of the search region—By shifting the region towards one of the sides orthogonal to the side it currently touches (namely, and w.l.o.g — eastwards if the region touches the northern or southern sides, and northwards for the eastern and western sides) a convex region with $P'(t) \le P(t)$ is obtained. Using the next section discussing regions touching two sides of the search region, it can be seen that $P'(t) > X$, and therefore $P(t) > X$. See an example in Fig. 4.

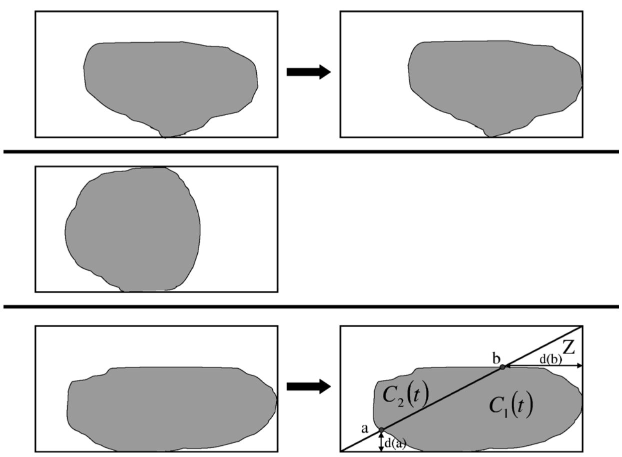

Fig. 4. An illustration of the second and third cases of Lemma 4.

- $C(t)$ touches two sides of the search region—In case the two sides are the north-south or east–west couples, it is clear that $P(t) \ge 2X$. For any other couple of region's sides, let us consider Z, the triangle formed when dissecting the search region diagonally in such a way that the sides touching $C(t)$ are both part of Z. Since the area of Z equals $\frac{XY}{2}$, then since $S(t) = \frac{XY}{2}$ as well, either $Z = C(t)$ (in which case $P(t) = \sqrt{X^2+Y^2} > X$) or all the following hold:
  - $C(t) = C_1(t) \cup C_2(t)$
  - $C_1(t) \cap C_2(t) = \emptyset$
  - $Z \supset C_1(t)$
  - $Z \cap C_2(t) = \emptyset$

  Let us assume without loss of generality that $C(t)$ touches the western and southern sides of the search region. Let us denote by a and b the two points along the hypotenuse of Z, which are contained in $C(t)$, and which are the closest to the western and southern sides, respectively. Let us denote the distance between the western side to a by $d(a)$ and the distance between the southern side to b by $d(b)$. It is clear that $P(t) \ge d(a) + d(b) + P'(t)$ (where $P'(t)$ denotes the perimeter of $C_2(t)$, excluding the part of $C_2(t)$ which touches Z). As $P'(t) > \overline{ab}$ (the subsegment of the hypotenuses of Z which touches $C_2(t)$, which is located exactly between a and b, due to the convexity of $C(t)$), we can see that $P(t)$ has at least the length of some path beginning at the western side of the search region, proceeding to the hypotenuses of Z (at point a), going along the hypotenuses of Z until reaching the point b, and proceeding to the southern side of the search region.

  We now show that the length of any path starting at the western side, going to the hypotenuses of Z, and returning to the southern side, is at least X.

  It is obvious that the shortest path of such qualities will be in the form of two straight lines, one starting at some point c along the western side, going straight eastwards until reaching the hypotenuses of Z (denoted by e), and another going from this point straight southwards (denoted by f). Let $d(e)$ denote the length of e and let $d(c)$ denote the distance between the western northern corner of the search region to point c. Note that $d(f) = X - d(c)$. The length of the path is therefore $P = d(e) + X - d(c)$. Note that since $\frac{d(c)}{d(e)} = \frac{X}{Y}$ the path's length can be written as $P = d(e) + \left(X - \frac{X}{Y}d(e)\right) = d(e)\left(1 - \frac{X}{Y}\right) + X$. It can easily be seen that for every value of $0 \le d(e) \le Y$ we see that $\frac{\partial P}{\partial d(e)} > 0$ (because $X \le Y$) and therefore the minimal value of P is produced for $d(e) = 0$, which is $P = X$. See an example in Fig. 4.
- $C(t)$ touches three sides of the search region—clearly, if the side not touched by $C(t)$ is the western or eastern, then $P(t) \ge X$ whereas if the side not touched by $C(t)$ is the northern or southern, then $P(t) \ge Y \ge X$.
- $C(t)$ touches all the four sides of the search region—Let k be the number of corners of the rectangle not contained in $C(t)$. Let a_i and b_i be the lengths of the portions of the sides adjacent to corner i that are not parts of $C(t)$ (a_i in the X coordinate and b_i in the Y coordinate). See an illustration of these notations in Fig. 4. Then the combined area of the triangles, formed by connecting these portions

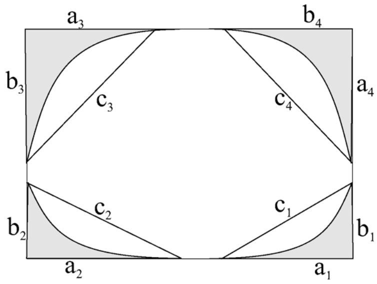

Fig. 5. An illustration of the last case of Lemma 4. The clean region in the center is $C(t)$.

of the region's sides, is

$$\sum_{i=1}^{k} \frac{a_i b_i}{2}.$$

As $C(t)$ is convex, by the definition of the triangles above, they contain all the area of the search region, which is not part of $C(t)$ (however, parts of the triangles may also be contained in $C(t)$). Therefore, the combined area of the triangles must be at least $\frac{XY}{2}$, and thus

$$\sum_{i=1}^{k} \frac{a_i b_i}{2} \geq \frac{XY}{2}. \tag{11}$$

Since $(a-b)^2 = a^2 + b^2 - 2ab$, and since $(a-b)^2 > 0$ for all $a, b \in \mathbb{R}$, we know that $a^2 + b^2 - 2ab > 0$ and therefore $a^2 + b^2 > 2ab$ and therefore $\frac{a^2+b^2}{2} > ab$. Hence

$$\sum_{i=1}^{k} \left(\frac{a_i^2 + b_i^2}{2} \right) \geq \sum_{i=1}^{k} a_i b_i \geq 2 \sum_{i=1}^{k} \frac{a_i b_i}{2} \geq XY. \tag{12}$$

and also

$$\sum_{i=1}^{k} \left(a_i^2 + b_i^2 \right) \geq 2XY \tag{13}$$

Denoting c_i the hypotenuses of the triangles, we get

$$\sum_{i=1}^{k} c_i^2 \geq 2XY \geq 2X^2. \tag{14}$$

As $(\sum_{i=1}^{k} c_i)^2 \geq \sum_{i=1}^{k} c_i^2$ we can write

$$\left(\sum_{i=1}^{k} c_i \right)^2 \geq 2X^2, \tag{15}$$

and therefore,

$$\sum_{i=1}^{k} c_i \geq \sqrt{2} X > X. \tag{16}$$

As the sum of the hypotenuses of the triangles is a lower bound over $P(t)$ (since it is a sum of straight lines between pairs of points), we have shown that $P(t) > X$. □

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