the potential flexibility of the system becomes illusory [28]. Alternatively he may move to the level of generating instructions for data processing, which, however, may involve him in a considerable episodic memory load in keeping track of system states from run to run. In designing parameter formats, therefore, attempts might be made to envisage and trace through the whole sequence of package use, checking that the use of one-shot commands or similar requirement is eliminated wherever possible. However, direct application of the results of a single experiment is hazardous: ergonomics requires knowledge of the range of conditions over which a given effect holds, and of the determinants of the relative magnitude of the effect. Two major immediate tasks for the psychology of commands will be to extend investigation beyond the processing of a single command to the handling of relationships among commands (e.g., in such operations as command decomposition and command inference), and to embrace such categorical commands as the "action primitives" that underly Treu's design proposals for interactive command language [30]. And in the same way that experimental studies of programming languages have investigated both comprehensibility [17], [23], [24], [32] and ease of use [13], [21], [32], [33], so the psychology of imperative statements must provide data on the generation, as well as the interpretation, of commands.

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A Note on Single Signed Integral Pulse Frequency Modulation

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Abstract—**The single signed integral pulse frequency modulation (SS-IPFM) is used in modeling neural communication processes, hence its significance. An expression equivalent to the SS-IPFM signal, suitable for spectral analysis is rederived here by the formalism used in the pulse position modulation (PPM) spectral analysis, thereby revealing some similarities between the two coding schemes. This result provides powerful means, readily available from communication theory, for the analysis of neural processes modeled by SS-IPFM.**

I. INTRODUCTION

Since the integral pulse frequency modulation (IPFM) was first proposed by Li and Meyer [1], [2], there has been interest in it as a model for neural encoding processes. Indeed, under certain conditions, the definition of the single signed IPFM (SS-IPFM) is an adequate mathematical representation of what physiologists believe to be the process of some transformations of graded potentials, such as receptor's generator potential, into sequences of nerve impulses [3], [4].

Let us consider a basic neural communications system modeled as follows: a SS-IPFM encoder, the axonal channel, and a low-pass filtering (LPF) decoder—accounting for some sort of averaging at the receiving end.

As in any communication system, the goal to be achieved here is reliable transmission of information in spite of encoder distortions and channel noise contaminating the signal [5]–[7]. In order to evaluate distortion and noise effects, spectral analysis is a widely used technique; therefore, it is not surprising that most of the analytical research on the SS-IPFM was devoted to its spectral characteristics.

Since communication theory does not provide standard methods for analysis of modulated sequences of pulses, the derivation of SS-IPFM spectrum was based either on some adaptation of techniques previously applied to other modulation schemes or on the invention of new, specific ones.

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By the first way, Bayly obtained an explicit expression for the spectrum of a single tone modulated sequence of rectangular pulses [8]. A new, specific method was devised by Lee [9]. Through the introduction of a "functional model" of the SS-IPFM encoder, he elegantly derived an expression for the modulated signal which is suitable for spectral analysis. Although his result is general and provides insight about the spectral characteristics of the signal, an explicit expression for its spectrum can, here too, be given only for a limited repertoire of input functions.

The purpose of this correspondence is to further increase the analytical power available for the study of neural communication system models employing the SS-IPFM scheme. This is achieved through an alternative derivation of Lee's expansion for the SS-IPFM signal by directly applying the formalism used in the spectral analysis of the naturally sampled pulse position modulation (PPM). Then an interesting similarity between the two modulation schemes (the PPM and SS-IPFM) becomes almost obvious from the formalism applied. Its significance with respect to the analysis of neural communication systems is pointed out and discussed.

II. DEFINITION OF SS-IPFM SIGNAL

Let $f(t) > 0$ be the modulating (input) function. The SS-IPFM signal $f^*(t)$ is defined as a train of impulses occurring at the times t_k,

$$f^*(t) = \sum_{k=-\infty}^{+\infty} \delta(t - t_k) \tag{1}$$

where the times of occurrence of consecutive impulses are related by

$$\int_{t_{k-1}}^{t_k} f(\zeta)\, d\zeta = A \tag{2}$$

and A represents the threshold of the encoder.

Note that if a train of pulses having a shape $p(t)$ is to be produced (instead of the impulse train) the output will be

$$f_p^*(t) = p(t) \circledast f^*(t) \tag{3}$$

where $\circledast$ denotes the convolution operator.

It is easily seen that if t_0 is defined to be the time origin ($t_0 = 0$), an equivalent expression for (2) is given by

$$\int_0^{t_k} f(\zeta)\, d\zeta = kA, \qquad \text{for all } k. \tag{4}$$

III. SPECTRAL ANALYSIS OF SS-IPFM SIGNAL

Using the "functional encoder" defined by him, Lee has shown [9] that for the purpose of spectral analysis one can write an equivalent expression for $f^*(t)$ as

$$f^*(t) = f(t)\left[\frac{1}{A} + \frac{2}{A}\sum_{n=1}^{\infty} \cos\left\{\frac{2\pi n}{A} \cdot \int_0^t f(\zeta)\, d\zeta\right\}\right]. \tag{5}$$

We shall give here an analytical derivation of (5) based on the formalism devised by Rowe for the analysis of the naturally sampled PPM [10]. Rowe's technique employs the following property of the Dirac delta function: if $g(t)$ is a function having a single, simple zero at λ ($g(\lambda) = 0$ and $g(t) \neq 0$ iff $t \neq \lambda$), then

$$\delta(t - \lambda) = \delta(g(t))\,|g'(t)|. \tag{6}$$

For a formal proof of (6) see [11].

For the purpose of derivation of Lee's expression (5), let us define a function $g(t,k)$ as

$$g(t,k) = \int_0^t f(\zeta)\, d\zeta - kA. \tag{7}$$

It is easy to check that $g(t,k)$ is a monotonically increasing function of t ($f(t) > 0$ by definition), $g(t,k) = 0$ having the unique solution at $t = t_k$ (see (4)). Furthermore, since $g'(t,k) = f(t)$ is positive, one can write (6) for this case as

$$\delta(t - t_k) = \delta(g(t,k))f(t). \tag{8}$$

Substituting (8) for $\delta(t - t_k)$ into (1) we obtain

$$f^*(t) = f(t) \sum_{k=-\infty}^{+\infty} \delta(g(t,k))$$

or

$$f^*(t) = f(t) \sum_{k=-\infty}^{+\infty} \delta\left(\int_0^t f(\zeta)\, d\zeta - kA\right). \tag{9}$$

Let us now define an auxiliary variable ρ as follows:

$$\rho \triangleq \int_0^t f(\zeta)\, d\zeta. \tag{10}$$

Using the Fourier expansion of a sequence of equally spaced (on the ρ axis) delta functions [12], one formally obtains

$$\sum_{k=-\infty}^{+\infty} \delta(\rho - kA) = \frac{1}{A} + \frac{2}{A}\sum_{n=1}^{+\infty} \cos\frac{2\pi n}{A}\rho. \tag{11}$$

Now substituting (10) for ρ in (11) we obtain

$$\sum_{k=-\infty}^{+\infty} \delta\left(\int_0^t f(\zeta)\, d\zeta - kA\right) = \frac{1}{A} + \frac{2}{A}\sum_{n=1}^{\infty} \cos\left\{\frac{2\pi n}{A}\int_0^t f(\zeta)\, d\zeta\right\}. \tag{12}$$

From (9) and (12) it follows immediately that

$$f^*(t) = f(t)\left[\frac{1}{A} + \frac{2}{A}\sum_{n=1}^{\infty} \cos\left\{\frac{2\pi n}{A}\int_0^t f(\zeta)\, d\zeta\right\}\right]$$

which is the required result stated in (5).

Thus, by applying the formalism of PPM spectral analysis to the SS-IPFM signal, we easily rederive the expression found by Lee. This fact shows that there might be an intrinsic similarity between the PPM and SS-IPFM. A careful look will show that in a way they are identical.

IV. COMPARISON OF THE PPM WITH SS-IPFM

Separating the input function $f(t) > 0$ into its dc and ac components (as is usually done):

$$f(t) = f_{\mathrm{dc}} + f_{\mathrm{ac}}(t)$$

and dividing $g(t,k)$ (defined in (7)) by f_{dc} we obtain

$$\bar{g}(t,k) = t - k\frac{A}{f_{\mathrm{dc}}} + \frac{1}{f_{\mathrm{dc}}}\int_0^t f_{\mathrm{ac}}(\zeta)\, d\zeta. \tag{13}$$

Using $\bar{g}(t,k)$ instead of $g(t,k)$, we obtain from (8) and (9) the following equivalent form for $f^*(t)$:

$$f^*_{\mathrm{SS\text{-}IPFM}}(t) = \left(1 + \frac{1}{f_{\mathrm{dc}}}f_{\mathrm{ac}}(t)\right) \cdot \left[\sum_{k=-\infty}^{+\infty} \delta\left(t - k\frac{A}{f_{\mathrm{dc}}} + \frac{1}{f_{\mathrm{dc}}}\int_0^t f_{\mathrm{ac}}(\zeta)\, d\zeta\right)\right]. \tag{14}$$

The counterpart of this expression in the naturally sampled PPM having a clock of period T and being modulated by an input function $m(t)$ is [10,, p. 262]:

$$f^*_{\mathrm{PPM}}(t) = (1 - m'(t)) \sum_{k=-\infty}^{+\infty} \delta(t - kT - m(t)), \tag{15}$$

where the derivative of the input $m'(t)$ is less than 1.

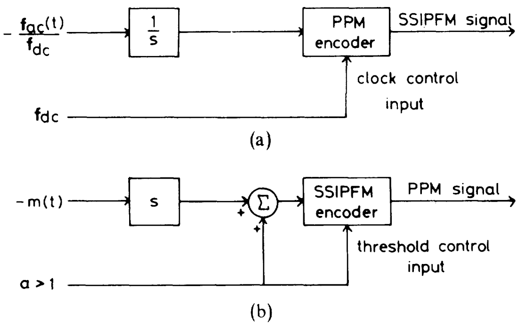

Fig. 1. (a) Schematic diagram that describes manner of obtaining SS-IPFM signal by means of PPM encoder. The PPM clock control input sets the period of the clock to $T = A/f_{dc}$. (b) Schematic diagram that describes manner of obtaining PPM signal by means of SS-IPFM encoder. A dc input $a > 1$, sets the threshold to $A = T \cdot a$.

Now comparing (14) and (15) it is apparent that if

$$T = \frac{A}{f_{dc}} \quad \text{and} \quad m(t) = -\frac{1}{f_{dc}} \int_0^t f_{ac}(\zeta)\, d\zeta,$$

then we obtain

$$f^*_{\text{PPM}}(t) \equiv f^*_{\text{SS-IPFM}}(t).$$

Note that because $f(t) > 0$, we have

$$m'(t) = -\frac{f_{ac}(t)}{f_{dc}} < 1$$

which satisfies the required condition for PPM. A schematic diagram illustrating this result is given in Fig. 1(a). The equivalence implies also that one could conceptually construct a scheme for PPM employing a SS-IPFM encoder. This is demonstrated in Fig. 1(b).

V. DISCUSSION

The equivalence established above makes it possible to apply a considerable amount of analytical knowledge from communication theory to the analysis of neural communication systems. In this context a similar result was obtained by Lee [9], who has shown the equivalence of SS-IPFM to continuous pulse frequency modulation. We note that a great deal of relevant research has been devoted to multiunit, multipath, neural communication, [8], [9], [13], [14], and to improved models. As such, the leaky-SS-IPFM was analyzed by Poppele and Chen [15], employing a PPM scheme. Thus it seems to us that the approach proposed here will enable a deeper understanding of the systems under study. In particular, problems such as the effects of adaptation, and stochastic fluctuations of threshold and membrane potential are currently under investigation.

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Increasing the Power of a Threshold Logic Unit by Using Binary Variables

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***Abstract*—Computer simulations show that mapping real variables to binary variables increases a threshold logic unit's classificatory power and thus increases the number of pattern recognition problems on which a threshold logic unit can be used successfully.**

Replacing one system of variables by another is a well-known technique for improving the performance of a pattern recognition system (e.g., discriminant analysis). Here we present several similar computer simulations which show that a threshold logic unit's (TLU) performance is improved when real variables are mapped to binary variables. Although a TLU always implements a linear decision boundary, the mapping causes an improvement in performance because a decision boundary linear in the derived system of binary variables may be highly nonlinear in the original system of real variables.

The mapping of real to binary variables is straightforward. Each binary variable corresponds to a threshold in a real variable's range and is one for a pattern if the real variable's value for the pattern is above the threshold, otherwise the binary variable is zero.

Although it is not the objective in [1] and [4] to demonstrate that mapping real variables to binary variables improves a classifier's recognition rate, [1] and [4] illustrate techniques for determining the number and placement of the thresholds needed for such a mapping. If the resulting system of binary variables contains redundant variables, these redundant variables can be eliminated by using a feature selection technique (as in [3], where Mucciardi's and Gose's [2] weighted sum feature selection technique is used).

The simulations described here are on the nonlinear two-class problem illustrated in Fig. 1(a). The symbols − and + denote patterns of class I and II, respectively. A system of binary variables is obtained very simply from the problem's system of two

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